Chapter
Ray Optics and Optical Instruments

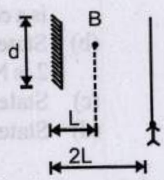
Topic-1: Plane Mirror, Spherical Mirror and Reflection of Light

1 MCQs with One Correct Answer

1. A ray of light travelling in the direction $\frac{1}{2}(\hat{i}+\sqrt{3}\hat{j})$ is incident on a plane mirror. After reflection, it travels along the direction $\frac{1}{2}(\hat{i}-\sqrt{3}\hat{j})$. The angle of incidence is **[Adv. 2013]**
(a) 30° (c) 60° (b) 45° (d) 75°

2. In an experiment to determine the focal length (f) of a concave mirror by the u - v method, a student places the object pin A on the principal axis at a distance x from the pole P. The student looks at the pin and its inverted image from a distance keeping his/her eye in line with PA. When the student shifts his/her eye towards left, the image appears to the right of the object pin. Then, **[2007]**
(a) $x < f$ (b) $f < x < 2f$
(c) $x = 2f$ (d) $x > 2f$

3. A point source of light B is placed at a distance L in front of the centre of a mirror of width 'd' hung vertically on a wall. A man walks in front of the mirror along a line parallel to the mirror at a distance $2L$ from it as shown in fig. The greatest distance over which he can see the image of the light source in the mirror is **[2000S]**



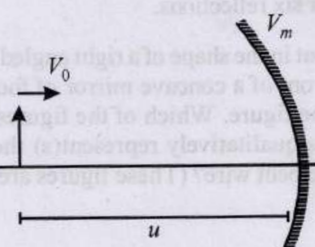
(a) $d/2$ (b) d (c) $2d$ (d) $3d$

4. A short linear object of length b lies along the axis of a concave mirror of focal length f at a distance u from the pole of the mirror. The size of the image is approximately equal to **[1988 - 2 Mark]**
(a) $b\left(\frac{u-f}{f}\right)^{1/2}$ (b) $b\left(\frac{f}{u-f}\right)^{1/2}$
(c) $b\left(\frac{u-f}{f}\right)$ (d) $b\left(\frac{f}{u-f}\right)^{2}$

2 Integer Value Answer

5. A person of height 1.6 m is walking away from a lamp post of height 4 m along a straight path on the flat ground. The lamp post and the person are always perpendicular to the ground. If the speed of the person is 60 cm s^{-1}, the speed of the tip of the person's shadow on the ground with respect to the person is _____ cm s^{-1}. **[Adv. 2023]**

6. An object and a concave mirror of focal length $f = 10\ cm$ both move along the principal axis of the mirror with constant speeds. The object moves with speed $V_0 = 15\ cm\ s^{-1}$ towards the mirror with respect to a laboratory frame. The distance between the object and the mirror at a given moment is denoted by u. When $u = 30\ cm$, the speed of the mirror V_m is such that the image is instantaneously at rest with respect to the laboratory frame, and the object forms a real image. The magnitude of V_m is _____ $cm\ s^{-1}$. **[Adv. 2022]**



3 Numeric / New Stem Based Questions

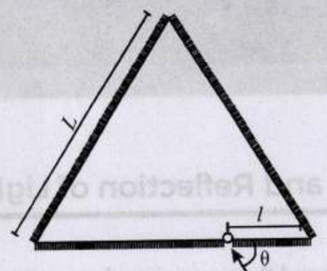
7. Image of an object approaching a convex mirror of radius of curvature 20 m along its optical axis is observed to move from $\frac{25}{3}$ m to $\frac{50}{7}$ m in 30 seconds. What is the speed of the object in km per hour? **[2010]**

4 Fill in the Blanks

8. A thin rod of length $\frac{f}{3}$ is placed along the optic axis of a concave mirror of focal length f such that its image which is real and elongated, just touches the rod. The magnification is .... **[1991 - 1 Mark]**

6 MCQs with One or More than One Correct Answer

9. Three plane mirrors from an equilateral triangle with each side of length L. There is a small hole at a distance $l > 0$ from one of the corners as shown in the figure. A ray of light is passed through the hole at an angle θ and can only come out through the same hole. The cross section of the mirror configuration and the ray of light lie on the same plane. **[Adv. 2022]**



Which of the following statement(s) is(are) correct?

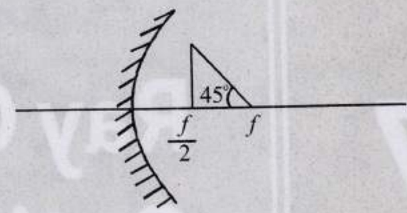
(a) The ray of light will come out for $\theta = 30°$, for $0 < l < L$.

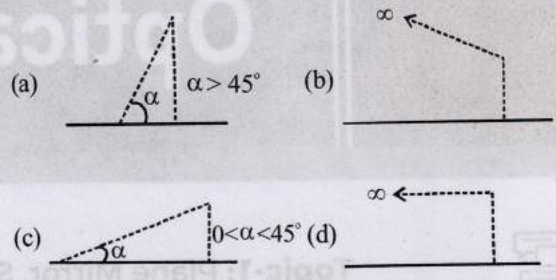
(b) There is an angle for $l = \frac{L}{2}$ at which the ray of light will come out after two reflections.

(c) The ray of light will **NEVER** come out for $\theta = 60°$, and $l = \frac{L}{3}$.

(d) The ray of light will come out for $\theta = 60°$, and $0 < l < \frac{L}{2}$ after six reflections.

10. A wire is bent in the shape of a right angled triangle and is placed in front of a concave mirror of focal length f, as shown in the figure. Which of the figures shown in the four options qualitatively represent(s) the shape of the image of the bent wire? (These figures are not to scale.) **[Adv. 2018]**





11. A student performed the experiment of determination of focal length of a concave mirror by u-v method using an optical bench of length 1.5 meter. The focal length of the mirror used is 24 cm. The maximum error in the location of the image can be 0.2 cm. The 5 sets of (u, v) values recorded by the student (in cm) are : (42, 56), (48, 48), (60, 40), (66, 33), (78, 39). The data set(s) that cannot come from experiment and is (are) incorrectly recorded, is (are) **[2009]**

(a) (42, 56) (b) (48, 48) (c) (66, 33) (d) (78, 39)

9 Assertion and Reason Type Questions

12. **STATEMENT-1** The formula connecting u, v and f for a spherical mirror is valid for mirrors whose sizes are very small compared to their radii of curvature.

STATEMENT-2 Laws of reflection are strictly valid for plane surfaces, but not for large spherical surfaces. **[2007]**

(a) Statement-1 is True, Statement-2 is True; Statement-2 is a correct explanation for Statement-1

(b) Statement-1 is True, Statement-2 is True; Statement-2 is NOT a correct explanation for Statement-1

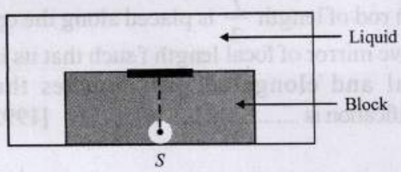
(c) Statement-1 is True, Statement-2 is False

(d) Statement-1 is False, Statement-2 is True

Topic-2: Refraction of Light at Plane Surface and Total Internal Reflection

1 MCQs with One Correct Answer

1. A point source S is placed at the bottom of a transparent block of height 10 mm and refractive index 2.72. It is immersed in a lower refractive index liquid as shown in the figure. It is found that the light emerging from the block to the liquid forms a circular bright spot of diameter 11.54 mm on the top of the block. The refractive index of the liquid is **[Adv. 2014]**



(a) 1.21 (b) 1.30

(c) 1.36 (d) 1.42

2. A ball is dropped from a height of 20 m above the surface of water in a lake. The refractive index of water is $\frac{4}{3}$. A fish inside the lake, in the line of fall of the ball, is looking at the ball. At an instant, when the ball is 12.8 m above the water surface, the fish sees the speed of ball as [Take $g = 10\ m/s^2$.] **[2009]**

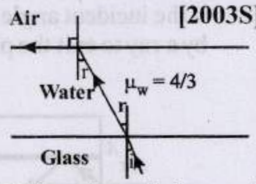
(a) 9 m/s (b) 12 m/s (c) 16 m/s (d) 21.33 m/s

3. A ray of light traveling in water is incident on its surface open to air. The angle of incidence is θ, which is less than the critical angle. Then there will be **[2007]**

(a) only a reflected ray and no refracted ray
(b) only a refracted ray and no reflected ray
(c) a reflected ray and a refracted ray and the angle between them would be less than $180° - 2\theta$
(d) a reflected ray and a refracted ray and the angle between them would be greater than $180° - 2\theta$

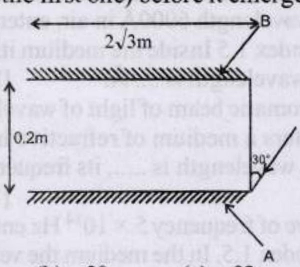
4. A point object is placed at the centre of a glass sphere of radius 6 cm and refractive index 1.5. The distance of virtual image from the surface is **[2004S]**
(a) 6 cm (b) 4 cm (c) 12 cm (d) 9 cm

5. A ray of light is incident at the glass-water interface at an angle i, it emerges finally parallel to the surface of water, then the value of μ_g would be **[2003S]**



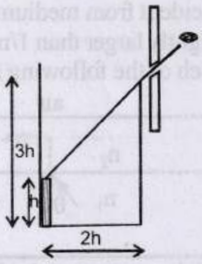
(a) $(4/3)\sin i$
(b) $1/\sin i$
(c) 4/3
(d) 1

6. Two plane mirrors A and B are aligned parallel to each other, as shown in the figure. A light ray is incident at an angle 30° at a point just inside one end of A. The plane of incidence coincides with the plane of the figure. The maximum number of times the ray undergoes reflections (including the first one) before it emerges out is **[2002S]**



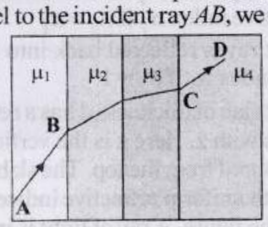
(a) 28 (b) 30 (c) 32 (d) 34

7. An observer can see through a pin-hole the top end of a thin rod of height h, placed as shown in the figure. The beaker height is 3h and its radius h. When the beaker is filled with a liquid up to a height 2h, he can see the lower end of the rod. Then the refractive index of the liquid is **[2002S]**



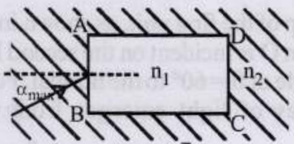
(a) $\frac{5}{2}$
(b) $\sqrt{\frac{5}{2}}$
(c) $\sqrt{\frac{3}{2}}$
(d) $\frac{3}{2}$

8. A ray of light passes through four transparent media with refractive indices μ_1, μ_2, μ_3 and μ_4 as shown in the figure. The surfaces of all media are parallel. If the emergent ray CD is parallel to the incident ray AB, we must have **[2001S]**



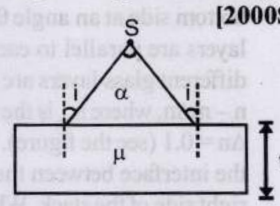
(a) $\mu_1 = \mu_2$ (b) $\mu_2 = \mu_3$ (c) $\mu_3 = \mu_4$ (d) $\mu_4 = \mu_1$

9. A rectangular glass slab $ABCD$ of refractive index n_1 is immersed in water of refractive index $n_2 (n_1 > n_2)$. A ray of light is incident at the surface AB of the slab as shown. The maximum value of the angle of incidence α_{max} such that the ray comes out only from the other surface CD is given by **[2000S]**



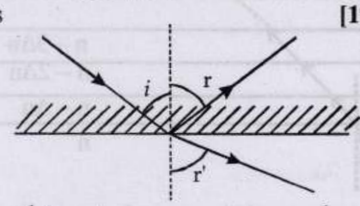
(a) $\sin^{-1}\left[\frac{n_1}{n_2}\cos\left(\sin^{-1}\left(\frac{n_2}{n_1}\right)\right)\right]$
(b) $\sin^{-1}\left[n_1\cos\left(\sin^{-1}\left(\frac{1}{n_2}\right)\right)\right]$
(c) $\sin^{-1}\left(\frac{n_1}{n_2}\right)$ (d) $\sin^{-1}\left(\frac{n_2}{n_1}\right)$

10. A diverging beam of light from a point source S having divergence angle α, falls symmetrically on a glass slab as shown. The angles of incidence of the two extreme rays are equal. If the thickness of the glass slab is t and the refractive index n, then the divergence angle of the emergent beam is **[2000S]**



(a) zero
(b) α
(c) $\sin^{-1}\left(\frac{1}{n}\right)$
(d) $2\sin^{-1}\left(\frac{1}{n}\right)$

11. A ray of light from a denser medium strike a rarer medium at an angle of incidence i (see Fig). The reflected and refracted rays make an angle of 90° with each other. The angles of reflection and refraction are r and r' The critical angle is **[1983 - 1 Mark]**

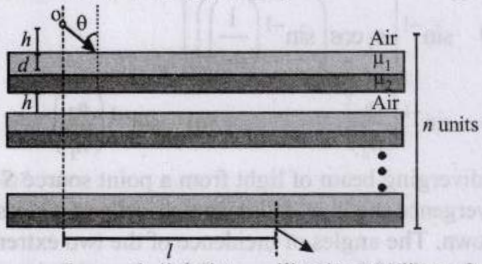


(a) $\sin^{-1}(\tan r)$ (b) $\sin^{-1}(\tan i)$
(c) $\sin^{-1}(\tan r')$ (d) $\tan^{-1}(\sin i)$

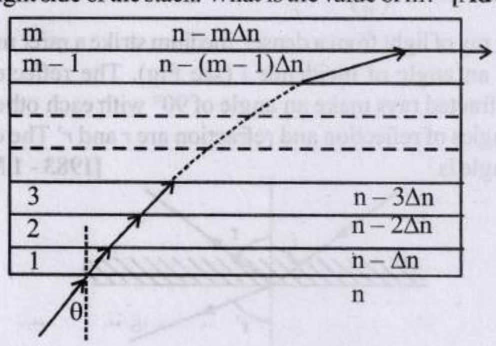
12. When a ray of light enters a glass slab from air, **[1980]**
(a) its wavelength decreases.
(b) its wavelength increases.
(c) Its frequency decreases.
(d) neither its wavelength nor its frequency changes.

2 Integer Value Answer

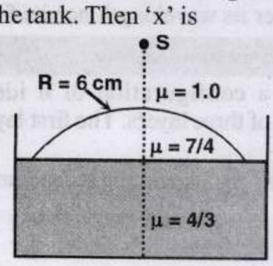
13. Consider a configuration of n identical units, each consisting of three layers. The first layer is a column of air of height $h = \frac{1}{3}\,cm$, and the second and third layers are of equal thickness $d = \frac{\sqrt{3}-1}{2}$ *cm*, and refractive indices $\mu_1 = \sqrt{\frac{3}{2}}$ and $\mu_2 = \sqrt{3}$, respectively. A light source O is placed on the top of the first unit, as shown in the figure. A ray of light from O is incident on the second layer of the first unit at an angle of $\theta = 60°$ to the normal. For a specific value of n, the ray of light emerges from the bottom of the configuration at a distance $l = \frac{8}{\sqrt{3}}$ *cm*, as shown in the figure. The value of n is ____. **[Adv. 2022]**



14. A monochromatic light is travelling in a medium of refractive index n = 1.6. It enters a stack of glass layers from the bottom side at an angle $\theta = 30°$. The interfaces of the glass layers are parallel to each other. The refractive indices of different glass layers are monotonically decreasing as $n_m = n - m\Delta n$, where n_m is the refractive index of the m^{th} slab and $\Delta n = 0.1$ (see the figure). The ray is refracted out parallel to the interface between the $(m-1)^{th}$ and m^{th} slabs from the right side of the stack. What is the value of m? **[Adv. 2017]**



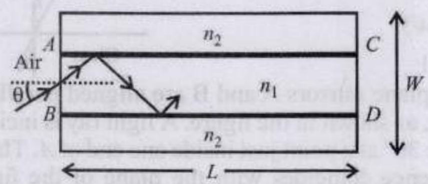
15. Water (with refractive index $= \frac{4}{3}$) in a tank is 18 cm deep. Oil of refractive index $\frac{7}{4}$ lies on water making a convex surface of radius of curvature 'R = 6 cm' as shown. Consider oil to act as a thin lens. An object 'S' is placed 24 cm above water surface. The location of its image is at 'x' cm above the bottom of the tank. Then 'x' is **[2011]**



16. A large glass slab ($\mu = 5/3$) of thickness 8 cm is placed over a point source of light on a plane surface. It is seen that light emerges out of the top surface of the slab from a circular area of radius R cm. What is the value of R? **[2010]**

3 Numeric / New Stem Based Questions

17. A planar structure of length L and width W is made of two different optical media of refractive indices $n_1 = 1.5$ and $n_2 = 1.44$ as shown in figure. If $L >> W$, a ray entering from end AB will emerge from end CD only if the total internal reflection condition is met inside the structure. For $L = 9.6 m$, if the incident angle θ is varied, the maximum time taken by a ray to exit the plane CD is $t \times 10^{-9} s$, where t is ____ **[Adv. 2019]**



4 Fill in the Blanks

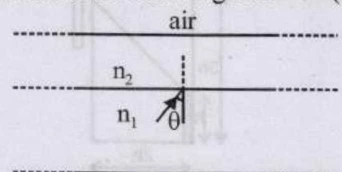
18. A light of wavelength 6000Å in air, enters a medium with refractive index 1.5 Inside the medium its frequency is .... Hz and its wavelength is .... Å. **[1997 - 2 Marks]**

19. A monochromatic beam of light of wavelength 6000 Å in vacuum enters a medium of refractive index 1.5. In the medium its wavelength is ....., its frequency is ...... **[1985 - 2 Marks]**

20. A light wave of frequency 5×10^{14} Hz enters a medium of refractive index 1.5. In the medium the velocity of the light wave is .... and its wavelength is .... **[1983 - 2 Marks]**

6 MCQs with One or More than One Correct Answer

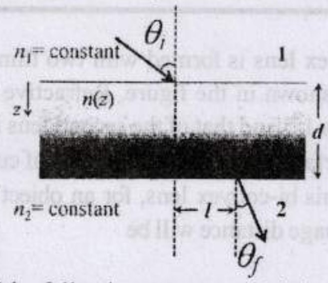
21. A wide slab consisting of two media of refractive indices n_1 and n_2 is placed in air as shown in the figure. A ray of light is incident from medium n_1 to n_2 at an angle θ, where sin θ is slightly larger than $1/n_1$. Take refractive index of air as 1. Which of the following statement(s) is(are) correct? **[Adv. 2021]**



(a) The light ray enters air if $n_2 = n_1$
(b) The light ray is finally reflected back into the medium of refractive index n_1 if $n_2 < n_1$
(c) The light ray is finally reflected back into the medium of refractive index n_1 if $n_2 > n_1$
(d) The light ray is reflected back into the medium of refractive index n_1 if $n_2 = 1$

22. A transparent slab of thickness d has a refractive index n(z) that increases with z. Here z is the vertical distance inside the slab, measured from the top. The slab is placed between two media with uniform refractive indices n_1 and n_2 ($> n_1$), as shown in the figure. A ray of light is incident with angle

θ_i, from medium 1 and emerges in medium 2 with refraction angle θ_f with a lateral displacement l. **[Adv. 2016]**



Which of the following statement(s) is(are) true?

(a) $n_1 \sin\theta_i = n_2 \sin\theta_f$
(b) $n_1 \sin\theta_i = (n_2 - n_1)\sin\theta_f$
(c) l is independent of n_2
(d) l is dependent on $n(z)$

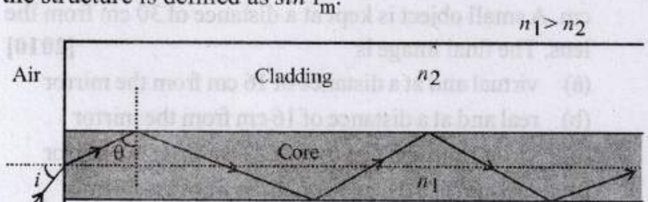
23. A ray of light travelling in a transparent medium falls on a surface separating the medium from air at an angle of incidence of 45°. The ray undergoes total internal reflection. If n is the refractive index of the medium with respect to air, select the possible value(s) of n from the following : **[1998 - 2 Marks]**

(a) 1.3 (b) 1.4 (c) 1.5 (d) 1.6

8 Comprehension/Passage Based Questions

Passage

Light guidance in an optical fibre can be understood by considering a structure comprising of thin solid glass cylinder of refractive index n_1 surrounded by a medium of lower refractive index n_2. The light guidance in the structure takes place due to successive total internal reflections at the interface of the media n_1 and n_2 as shown in the figure. All rays with the angle of incidence i less than a particular value i_m are confined in the medium of refractive index n_1. The numerical aperture (NA) of the structure is defined as $\sin i_m$.



24. For two structure namely S_1 with $n_1 = \sqrt{45}/4$ and $n_2 = 3/2$, and S_2 with $n_1 = 8/5$ and $n_2 = 7/5$ and taking the refractive index of water to be 4/3 and that of air to be 1, the correct option(s) is(are) **[Adv. 2015]**

(a) NA of S_1 immersed in water is the same as that of S_2 immersed in a liquid of refractive index $\frac{16}{3\sqrt{15}}$

(b) NA of S_1 immersed in liquid of refractive index $\frac{6}{\sqrt{15}}$ is the same as that of S_2 immersed in water

(c) NA of S_1 placed in air is the same as that of S_2 immersed in liquid of refractive index $\frac{4}{\sqrt{15}}$

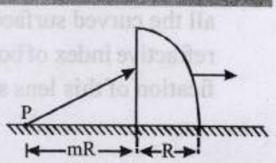
(d) NA of S_1 placed in air is the same as that of S_2 placed in water

25. If two structure of same cross-sectional area, but different numerical apertures NA_1 and NA_2 ($NA_2 < NA_1$) are joined longitudinally, the numerical aperture of the combined structure is **[Adv. 2015]**

(a) $\frac{NA_1 \, NA_2}{NA_1 + NA_2}$ (b) $NA_1 + NA_2$
(c) NA_1 (d) NA_2

10 Subjective Problems

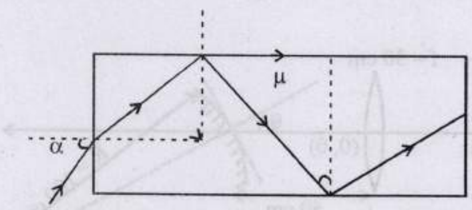
26. A quarter cylinder of radius R and refractive index 1.5 is placed on a table. A point object P is kept at a distance of mR from it.



Find the value of m for which a ray from P will emerge parallel to the table as shown in Figure. **[1999 - 5 Marks]**

27. The $x - y$ plane is the boundary between two transparent media. Medium -1 with $z \geq 0$ has a refractive index $\sqrt{2}$ and medium -2 with $z \leq 0$ has a refractive index $\sqrt{3}$. A ray of light in medium -1 given by the vector $A = 6\sqrt{3}\hat{i} + 8\sqrt{3}\hat{j} - 10\,\mathbf{k}$ is incident on the plane of separation. Find the unit vector in the direction of the refracted ray in medium-2. **[1999 - 10 Marks]**

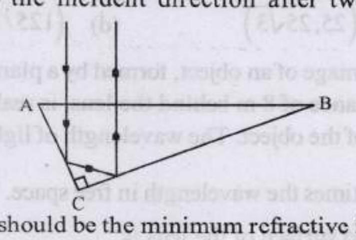
28. Light is incident at an angle α on one planar end of a transparent cylindrical rod of refractive index μ. Determine the least value of μ so that the light entering the rod does not emerge from the curved surface of rod irrespective of the value of α **[1992 - 8 Marks]**



29. A parallel beam of light travelling in water (refractive index = 4/3) is refracted by a spherical air bubble of radius 2 mm situated in water. Assuming the light rays to be paraxial **[1988 - 6 Marks]**

(i) Find the position of the image due to refraction at the first surface and the position of the final image.
(ii) Draw a ray diagram showing the positions of both the images.

30. A right prism is to be made by selecting a proper material and the angles A and B ($B \leq A$), as shown in Figure. It is desired that a ray of light incident on the face AB emerges parallel to the incident direction after two internal reflections.

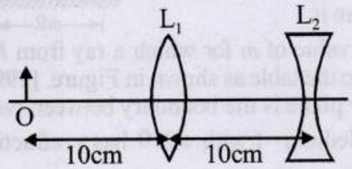


(i) What should be the minimum refractive index n for this to be possible ?
(ii) For $n = \frac{5}{3}$ is it possible to achieve this with the angle B equal to 30 degrees ? **[1987 - 7 Marks]**

Topic-3: Refraction at Curved Surface, Lenses and Power of Lens

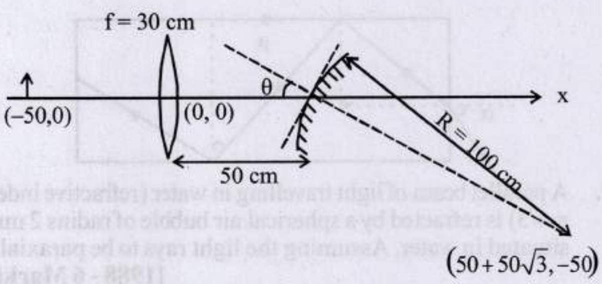
1 MCQs with One Correct Answer

1. An extended object is placed at point O, 10 cm in front of a convex lens L_1 and a concave lens L_2 is placed 10 cm behind it, as shown in the figure. The radii of curvature of all the curved surfaces in both the lenses are 20 cm. The refractive index of both the lenses is 1.5. The total magnification of this lens system is **[Adv. 2021]**



(a) 0.4 (b) 0.8
(c) 1.3 (d) 1.6

2. A small object is placed 50 cm to the left of a thin convex lens of focal length 30 cm. A convex spherical mirror of radius of curvature 100 cm is placed to the right of the lens at a distance of 50 cm. The mirror is tilted such that the axis of the mirror is at an angle θ = 30° to the axis of the lens, as shown in the figure.



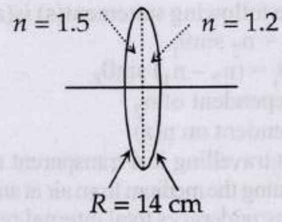
If the origin of the coordinate system is taken to be at the centre of the lens, the coordinates (in cm) of the point (x, y) at which the image is formed are **[Adv. 2016]**

(a) (0,0) (b) $(50-25\sqrt{3},25)$
(c) $(25,25\sqrt{3})$ (d) $(125/3,25\sqrt{3})$

3. The image of an object, formed by a plano-convex lens at a distance of 8 m behind the lens, is real is one-third the size of the object. The wavelength of light inside the lens is $\frac{2}{3}$ times the wavelength in free space. The radius of the curved surface of the lens is **[Adv. 2013]**

(a) 1m (b) 2m
(c) 3m (d) 6m

4. A bi-convex lens is formed with two thin plano-convex lenses as shown in the figure. Refractive index n of the first lens is 1.5 and that of the second lens is 1.2. Both the curved surface are of the same radius of curvature $R = 14$ cm. For this bi-convex lens, for an object distance of 40 cm, the image distance will be **[2012]**

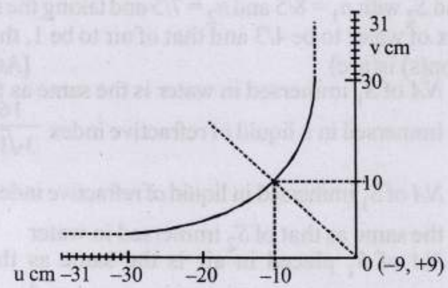


(a) -280.0 cm (b) 40.0 cm
(c) 21.5 cm (d) 13.3 cm

5. Rays of light from Sun falls on a biconvex lens of focal length f and the circular image of Sun of radius r is formed on the focal plane of the lens. Then **[2007]**
(a) Area of image is πr^2 and area is directly proportional of f
(b) Area of image is πr^2 and area is directly proportional to f^2
(c) Intensity of image increases if f is increased
(d) If lower half of the lens is covered with black paper area will become half

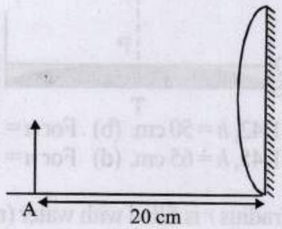
6. A biconvex lens of focal length 15 cm is in front of a plane mirror. The distance between the lens and the mirror is 10 cm. A small object is kept at a distance of 30 cm from the lens. The final image is **[2010]**
(a) virtual and at a distance of 16 cm from the mirror
(b) real and at a distance of 16 cm from the mirror
(c) virtual and at a distance of 20 cm from the mirror
(d) real and at a distance of 20 cm from the mirror

7. The graph shows relationship between object distance and image distance for a equiconvex lens. Then, focal length of the lens is **[2006 - 3M, –]]**



(a) 0.50 ± 0.05 cm (b) 0.50 ± 0.10 cm
(c) 5.00 ± 0.05 cm (d) 5.00 ± 0.10 cm

8. Focal length of the plano-convex lens is 15 cm. A small object is placed at A as shown in the figure. The plane surface is silvered. The image will form at **[2006 - 3M, –1]**



(a) 60 cm to the left of lens
(b) 12 cm to the left of lens
(c) 60 cm to the right of lens
(d) 30 cm to the left of lens

9. A convex lens is in contact with concave lens. The magnitude of the ratio of their focal length is 2/3. Their equivalent focal length is 30 cm. What are their individual focal lengths? **[2005S]**
(a) –15, 10 (b) –10, 15 (c) 75, 50 (d) –75, 50

10. The size of the image of an object, which is at infinity, as formed by a convex lens of focal length 30 cm is 2 cm. If a concave lens of focal length 20 cm is placed between the convex lens and the image at a distance of 26 cm from the convex lens, calculate the new size of the image. **[2003S]**
(a) 1.25 cm (b) 2.5 cm (c) 1.05 cm (d) 2 cm

11. A hollow double concave lens is made of very thin transparent material. It can be filled with air or either of two liquids L_1 or L_2 having refractive indices μ_1 and μ_2 respectively ($\mu_2 > \mu_1 > 1$). The lens will diverge a parallel beam of light if it is filled with **[2000S]**
(a) air and placed in air
(b) air and immersed in L_1
(c) L_1 and immersed in L_2
(d) L_2 and immersed in L_1

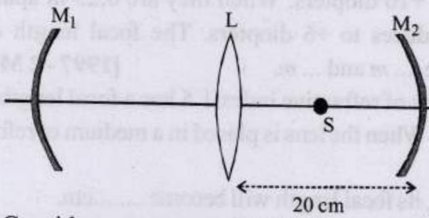
12. A concave lens of glass, refractive index 1.5 has both surfaces of same radius of curvature R. On immersion in a medium of refractive index 1.75, it will behave as a **[1999S - 2 Marks]**
(a) convergent lens of focal length 3.5 R
(b) convergent lens of focal length 3.0 R
(c) divergent lens of focal length 3.5 R
(d) divergent lens of focal length 3.0 R

13. A diminished image of an object is to be obtained on a screen 1.0 m from it. This can be achieved by appropriately placing **[1995S]**
(a) a concave mirror of suitable focal length
(b) a convex mirror of suitable focal length
(c) a convex lens of focal length less than 0.25 m
(d) a concave lens of suitable focal length

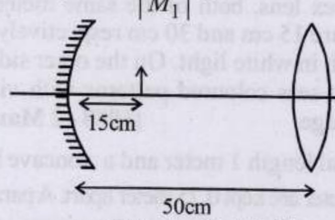
14. Spherical aberration in a thin lens can be reduced by **[1994 - 1 Mark]**
(a) using a monochromatic light
(b) using a doublet combination
(c) using a circular annular mark over the lens
(d) increasing the size of the lens.

2 Integer Value Answer

15. An optical arrangement consists of two concave mirrors M_1 and M_2, and a convex lens L with a common principal axis, as shown in the figure. The focal length of L is 10 cm. The radii of curvature of M_1 and M_2 are 20 cm and 24 cm, respectively. The distance between L and M_2 is 20 cm. A point object S is placed at the mid–point between L and M_2 on the axis. When the distance between L and M_1 is n/7 cm, one of the images coincides with S. The value of n is ______. **[Adv. 2023]**



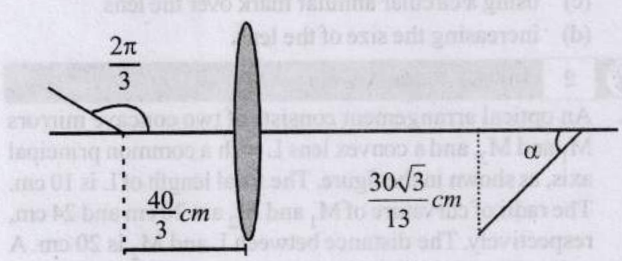
16. Consider a concave mirror and a convex lens (refractive index = 1.5) of focal length 10 cm each, separated by a distance of 50 cm in air (refractive index = 1) as shown in the figure. An object is placed at a distance of 15 cm from the mirror. Its erect image formed by this combination has magnification M_1. When the set-up is kept in a medium of refractive index $\frac{7}{6}$, the magnification becomes M_2. The magnitude $\left|\frac{M_2}{M_1}\right|$ is **[Adv. 2015]**



17. The focal length of a thin biconvex lens is 20 cm. When an object is moved from a distance of 25 cm in front of it to 50 cm, the magnification of its image changes from m_{25} to m_{50}. The ratio $\frac{m_{25}}{m_{50}}$ is **[2010]**

18. A rod of length 2 cm makes an angle $\frac{2\pi}{3}$ *rad* with the principal axis of a thin convex lens. The lens has a focal length of 10 cm and is placed at a distance of $\frac{40}{3}$ *cm* from the object as shown in the figure. The height of the image is $\frac{30\sqrt{3}}{13}$ *cm* and the angle made by it with respect to the

principal axis is α *rad*. The value of α is $\frac{\pi}{n}$ *rad*, where n is ______. **[Adv. 2022]**



4 Fill in the Blanks

19. Two thin lenses, when in contact, produce a combination of power +10 diopters. When they are 0.25 m apart, the power reduces to +6 diopters. The focal length of the lenses are .... *m* and ... *m*. **[1997 - 2 Marks]**

20. A thin lens of refractive index 1.5 has a focal length of 15 cm in air. When the lens is placed in a medium of refractive index $\frac{4}{3}$, its focal length will become ....cm. **[1987 - 2 Marks]**

21. A convex lens *A* of focal length 20 cm and a concave lens *B* of focal length 5 cm are kept along the same axis with a distance *d* between them. If a parallel beam of light falling on *A* leaves *B* as a parallel beam, then *d* is equal to ...... cm. **[1985 - 2 Marks]**

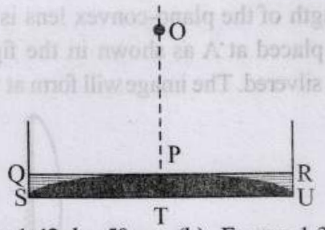
5 True / False

22. A parallel beam of white light fall on a combination of a concave and a convex lens, both of the same meterial. Their focal lengths are 15 cm and 30 cm respectively for the mean wavelength in white light. On the other side of the lens system, one sees coloured patterns with violet colour at the outer edge. **[1988 - 2 Marks]**

23. A convex lens of focal length 1 meter and a concave lens of focal length 0.25 meter are kept 0.75 meter apart. A parallel beam of light first passes through the convex lens, then through the concave lens and comes to a focus 0.5 m away from the concave lens. **[1983 - 2 Marks]**

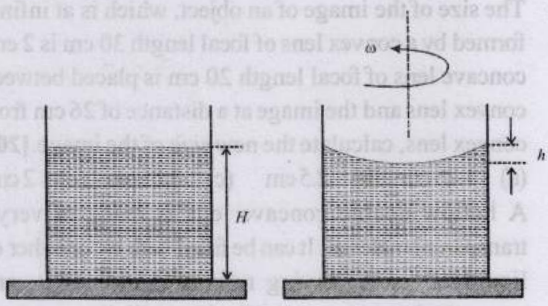
6 MCQs with One or More than One Correct Answer

24. A glass beaker has a solid, plano-convex base of refractive index 1.60, as shown in the figure. The radius of curvature of the convex surface (SPU) is 9 cm, while the planar surface (STU) acts as a mirror. This beaker is filled with a liquid of refractive index n up to the level QPR. If the image of a point object O at a height of h (OT in the figure) is formed onto itself, then, which of the following option(s) is(are) correct? **[Adv. 2024]**



(a) For $n = 1.42, h = 50$ cm. (b) For $n = 1.35, h = 36$ cm.
(c) For $n = 1.45, h = 65$ cm. (d) For $n = 1.48, h = 85$ cm.

25. A beaker of radius r is filled with water (refractive index $\frac{4}{3}$) up to a height H as shown in the figure on the left. The beaker is kept on a horizontal table rotating with angular speed ω. This makes the water surface curved so that the difference in the height of water level at the center and at the circumference of the beaker is $h (h \ll H, h \ll r)$, as shown in the figure on the right. Take this surface to be approximately spherical with a radius of curvature R. Which of the following is/are correct? (g is the acceleration due to gravity)



(a) $R = \frac{h^2 + r^2}{2h}$

(b) $R = \frac{3r^2}{2h}$

(c) Apparent depth of the bottom of the beaker is close to $\frac{3H}{2}\left(1 + \frac{\omega^2 H}{2g}\right)^{-1}$

(d) Apparent depth of the bottom of the beaker is close to $\frac{3H}{4}\left(1 + \frac{\omega^2 H}{4g}\right)^{-1}$

26. A plano-convex lens is made of a material of refractive index n. When a small object is placed 30 cm away in front of the curved surface of the lens, an image of double the size of the object is produced. Due to reflection from the convex surface of the lens, another faint image is observed at a distance of 10 cm away from the lens. Which of the following statement(s) is(are) true? **[Adv. 2016]**

(a) The refractive index of the lens is 2.5
(b) The radius of curvature of the convex surface is 45 cm
(c) The faint image is erect and real
(d) The focal length of the lens is 20 cm

27. A spherical surface of radius of curvature R separates air (refractive index 1.0) from glass (refractive index 1.5). The centre of curvature is in the glass. A point object P placed in air is found to have a real image *Q* in the glass. The line *PQ* cuts the surface at a point *O*, and *PO* = *OQ*. The distance *PO* is equal to **[1998 - 2 Marks]**

(a) 5*R* (b) 3*R*
(c) 2*R* (d) 1.5*R*

28. A real image of a distant object is formed by a plano-convex lens on its principal axis. Spherical aberration

(a) is absent. **[1998 - 2 Marks]**
(b) is smaller if the curved surface of the lens faces the object.
(c) is smaller if the plane surface of the lens faces the object.
(d) is the same whichever side of the lens faces the object

29. Which of the following form(s) a virtual and erect image for all positions of the object ? **[1996 - 2 Marks]**

(a) Convex lens (b) Concave lens
(c) Convex mirror (d) Concave mirror.

30. A converging lens is used to form an image on a screen. When the upper half of the lens is covered by an opaque screen **[1986 - 2 Marks]**

(a) half the image will disappear.
(b) complete image will be formed.
(c) intensity of the image will increase.
(d) intensity of the image will decrease.

31. A convex lens of focal length 40 cm is in contact with a concave lens of focal length 25 cm . The power of the combination is **[1982 - 3 Marks]**

(a) – 1.5 dioptres (b) – 6.5 dioptres
(c) + 6.5 dioptres (d) + 6.67 dioptres

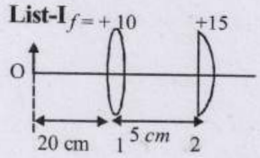
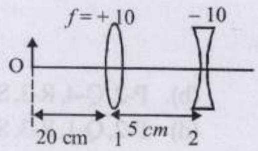
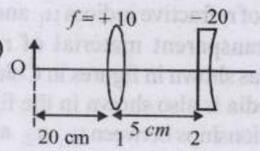
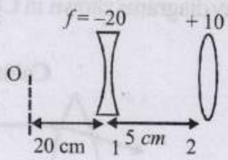
7 Match the Following

32. A light ray is incident on the surface of a sphere of refractive index *n* at an angle of incidence θ_0. The ray partially refracts into the sphere with angle of refraction ϕ_0 and then partly reflects from the back surface. The reflected ray then emerges out of the sphere after a partial refraction. The total angle of deviation of the emergent ray with respect to the incident ray is α. Match the quantities mentioned in List-I with their values in List-II and choose the correct option. **[Adv. 2024]**

List-I	List-II
(P) If $n = 2$ and $\alpha = 180°$, then all the possible values of θ_0 will be	(1) 30° and 0°
(Q) If $n = \sqrt{3}$ and $\alpha = 180°$, then all the possible values of θ_0 will be	(2) 60° and 0°
(R) If $n = \sqrt{3}$ and $\alpha = 180°$, then all the possible values of ϕ_0 will be	(3) 45° and 0°
(S) If $n = \sqrt{2}$ and $\theta_0 = 45°$, then all the possible values of α will be	(4) 150°
	(5) 0°

(a) P → 5; Q → 2; R → 1; S → 4
(b) P → 5; Q → 1; R → 2; S → 4
(c) P → 3; Q → 2; R → 1; S → 4
(d) P → 3; Q → 1; R → 2; S → 5

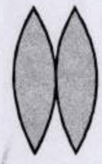
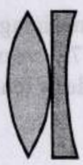
33. List I contains four combinations of two lenses (1 and 2) whose focal lengths (in *cm*) are indicated in the figures. In all cases, the object is placed 20 *cm* from the first lens on the left, and the distance between the two lenses is 5 *cm*. List II contains the positions of the final images. **[Adv. 2022]**

List-I	List-II
(I) 	(P) Final image is formed at 7.5 *cm* on the right side of lens 2.
(II) 	(Q) Final image is formed at 60.0 *cm* on the right side of lens 2.
(III) 	(R) Final image is formed at 30.0 *cm* on the left side of lens 2.
(IV) 	(S) Final image is formed at 6.0 *cm* on the right side of lens 2.
	(T) Final image is formed at 30.0 *cm* on the right side of lens 2.

Which one of the following options is correct?

(a) (I) → (P); (II) → (R); (III) → (Q); (IV) → (T)
(b) (I) → (Q); (II) → (P); (III) → (T); (IV) → (S)
(c) (I) → (P); (II) → (T); (III) → (R); (IV) → (Q)
(d) (I) → (T); (II) → (S); (III) → (Q); (IV) → (R)

34. Four combinations of two thin lenses are given in List-I. The radius of curvature of all curved surfaces is *r* and the refractive index of all the lenses is 1.5. Match lens combinations in List-I with their focal length in List-II and select the correct answer using the code given below the lists. **[Adv. 2014]**

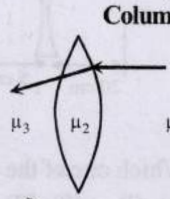
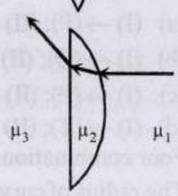
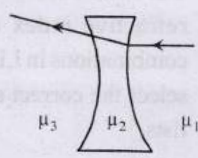
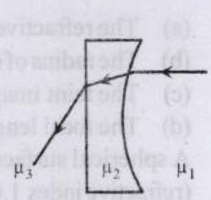
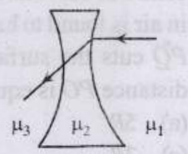
List - I		List - II
P.		**1.** $2r$
Q.		**2.** $\frac{r}{2}$
R.		**3.** $-r$
S.		**4.** r

Codes:

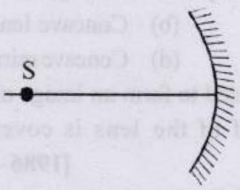
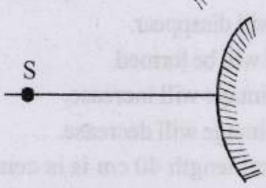
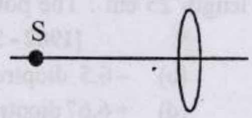
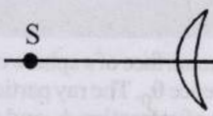
(a) P-1, Q-2, R-3, S-4 (b) P-2, Q-4, R-3, S-1

(c) P-4, Q-1, R-2, S-3 (d) P-2, Q-1, R-3, S-4

35. Two transparent media of refractive indices μ_1 and μ_3 have a solid lens shaped transparent material of refractive index μ_2 between them as shown in figures in Column II. A ray traversing these media is also shown in the figures. In Column I different relationships between μ_1, μ_2, and μ_3 are given. Match them to the ray diagrams shown in **Column II.** **[2010]**

Column I	Column II
(A) $\mu_1 < \mu_2$	(p) 
(B) $\mu_1 > \mu_2$	(q) 
(C) $\mu_2 = \mu_3$	(r) 
(D) $\mu_2 > \mu_3$	(s) 
	(t) 

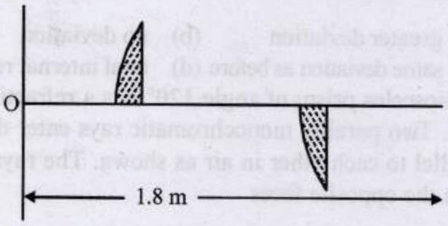
36. An optical component and an object S placed along its optic axis are given in Column I. The distance between the object and the component can be varied. The properties of images are given in Column II. Match all the properties of images from Column II with the appropriate components given in Column I. Indicate your answer by darkening the appropriate bubbles of the 4 × 4 matrix given in the ORS. **[2008]**

Column I	Column II
(A) 	(p) real image
(B) 	(q) virtual image
(C) 	(r) magnified image
(D) 	(s) image at infinity

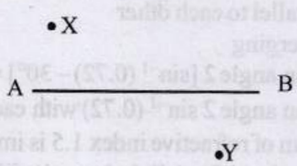
10 Subjective Problems

37. An object is moving with velocity 0.01 m/s towards a convex lens of focal length 0.3 m. Find the magnitude of rate of separation of image from the lens when the object is at a distance of 0.4 m from the lens. Also calculate the magnitude of the rate of change of the lateral magnification. **[2004 - 4 Marks]**

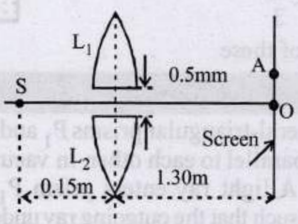
38. A thin plano-convex lens of focal length f is split into two halves: one of the halves is shifted along the optical axis. The separation between object and image planes is 1.8 m. The magnification of the image formed by one of the half-lenses is 2. Find the focal-length of the lens and separation between the two halves. Draw the ray diagram for image formation. **[1996 - 5 Marks]**



39. An image Y is formed of point object X by a lens whose optic axis is AB as shown in figure. Draw a ray diagram to locate the lens and its focus. If the image Y of the object X is formed by a concave mirror (Having the same axis as AB) instead of lens, draw another ray diagram to locate the mirror and its focus. Write down the steps of construction of the ray diagrams. **[1994 - 6 Marks]**



40. In Fig., S is a monochromatic point source emitting light of wavelength $\lambda = 500$nm. A thin lens of circular shape and focal length 0.10 m is cut into two identical halves L_1 and L_2 by a plane passing through a diameter. The two halves are placed symmetrically about the central axis SO with a gap of 0.5 mm. The distance along the axis from S to L_1 and L_2 is 0.15 m while that from L_1 and L_2 to O is 1.30 m. The screen at O is normal to SO. **[1993 - 5+1 Marks]**



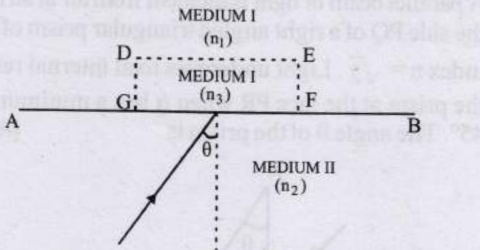
(i) If the third intensity maximum occurs at the point A on the screen, find the distance OA.

(ii) If the gap between L_1 and L_2 is reduced from its original value of 0.5mm, will the distance OA increase, decrease, or remain the same?

41. Monochromatic light is incident on a plane interface AB between two media of refractive indices n_1 and n_2 ($n_2 > n_1$) at an angle of incidence θ as shown in fig. The angle θ is infinitesimally greater than the critical angle for the two media so that total internal reflection takes place. Now if a transparent slab $DEFG$ of uniform thickness and of refractive index n_3 is introduced on the interface (as shown in the figure), show that for any value of n_3 all light will ultimately be reflected back again into medium II. Consider separately the cases **[1986 - 6 Marks]**

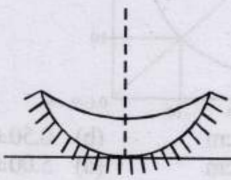
(i) $n_3 < n_1$ and

(ii) $n_3 > n_1$.



42. A plano convex lens has a thickness of 4 cm. When placed on a horizontal table, with the curved surface in contact with it, the apparent depth of the bottom most point of the lens is found to be 3 cm. If the lens is inverted such that the plane face is in contact with the table, the apparent depth of the centre of the plane face is found to be 25/8 cm. Find the focal length of the lens. **[1984- 6 Marks]**

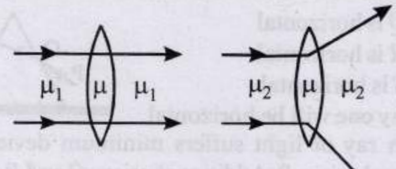
43. The convex surface of a thin concavo-convex lens of glass of refractive index 1.5 has a radius of curvature 20 cm. The concave surface has a radius of curvature 60 cm. The convex side is silvered and placed on a horizontal surface. **[1981- 6 Marks]**



(i) Where should a pin be placed on the optic axis such that its image is formed at the same place?

(ii) If the concave part is filled with water of refractive index 4/3, find the distance through which the pin should be moved so that the image of the pin again coincides with the pin.

44. What is the relation between the refractive indices μ_1 and μ_2, if the behaviour of light rays is as shown in the figure? **[1979]**

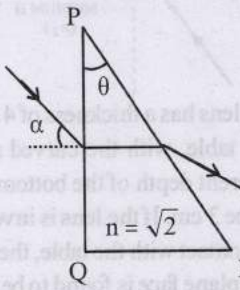


45. A pin is placed 10 cm in front of a convex lens of focal length 20 cm, made a material of refractive index 1.5. The surface of the lens farther away from the pin is silvered and has a radius of curvature are of 22 cm. Determine the position of the final image. Is the image real as virtual? **[1978]**

Topic-4: Prism and Dispersion of Light

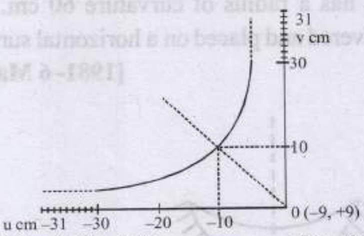
1 MCQs with One Correct Answer

1. A parallel beam of light is incident from air at an angle α on the side PQ of a right angled triangular prism of refractive index n = $\sqrt{2}$. Light undergoes total internal reflection in the prism at the face PR when α has a minimum value of 45°. The angle θ of the prism is **[Adv. 2016]**



(a) 15° (b) 22.5° (c) 30° (d) 45°

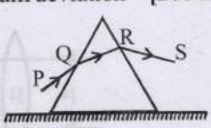
2. The graph shows relationship between object distance and image distance for a equiconvex lens. Then, focal length of the lens is **(2006 - 3M, –1)**



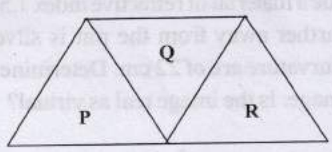
(a) 0.50±0.05 cm (b) 0.50±0.10 cm
(c) 5.00±0.05 cm (d) 5.00±0.10 cm

3. Two beams of red and violet colours are made to pass separately through a prism (angle of the prism is 60°). In the position of minimum deviation, the angle of refraction will be **[2008]**
(a) 30° for both the colours
(b) greater for the violet colour
(c) greater for the red colour
(d) equal but not 30° for both the colours

4. An equilateral prism is placed on a horizontal surface. A ray *PQ* is incident onto it. For minimum deviation **[2004S]**
(a) *PQ* is horizontal
(b) *QR* is horizontal
(c) *RS* is horizontal
(d) Any one will be horizontal

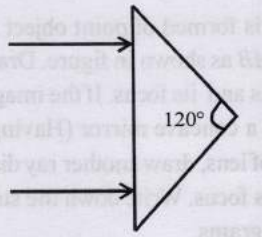


5. A given ray of light suffers minimum deviation in an equilateral prism *P*. Additional prism *Q* and *R* of identical shape and of the same material as *P* are now added as shown in the figure. The ray will now suffer **[2001S]**



(a) greater deviation (b) no deviation
(c) same deviation as before (d) total internal reflection

6. An isosceles prism of angle 120° has a refractive index 1.44. Two parallel monochromatic rays enter the prism parallel to each other in air as shown. The rays emerge from the opposite faces **[1995S]**



(a) are parallel to each other
(b) are diverging
(c) make an angle 2 [$\sin^{-1}(0.72) - 30°$] with each other
(d) make an angle 2 $\sin^{-1}(0.72)$ with each other

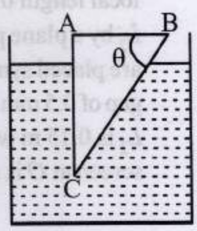
7. A glass prism of refractive index 1.5 is immersed in water (refractive index 4/3). A light beam incident normally on the face *AB* is totally reflected to reach on the face *BC* if **[1981-2 Marks]**

(a) $\sin\theta > \frac{8}{9}$

(b) $\frac{2}{3} < \sin\theta < \frac{8}{9}$

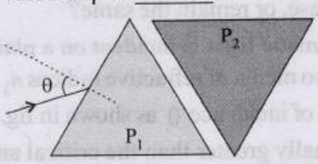
(c) $\sin\theta < \frac{2}{3}$

(d) None of these



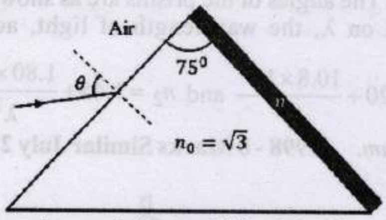
2 Integer Value Answer

8. Two equilateral-triangular prisms P_1 and P_2 are kept with their sides parallel to each other, in vacuum, as shown in the figure. A light ray enters prism P_1 at an angle of incidence θ such that the outgoing ray undergoes minimum deviation in prism P_2. If the respective refractive indices of P_1 and P_2 are $\sqrt{\frac{3}{2}}$ and $\sqrt{3}$, then $\theta = \sin^{-1}\left[\sqrt{\frac{3}{2}}\sin\left(\frac{\pi}{\beta}\right)\right]$, where the value of β is _____. **[Adv. 2024]**

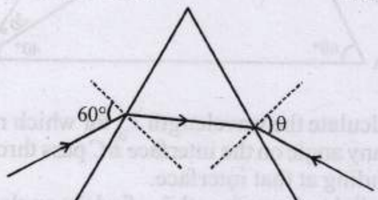


9. A monochromatic light is incident from air on a refracting surface of a prism of angle 75° and refractive index $n_0 = \sqrt{3}$. The other refracting surface of the prism is coated by a thin film of material of refractive index *n* as shown in figure. The light suffers total internal reflection

at the coated prism surface for an incidence angle of $\theta \leq 60^0$. The value of n^2 is **[Adv. 2019]**



10. The monochromatic beam of light is incident at 60° on one face of an equilateral prism of refractive index n and emerges from the opposite face making an angle $\theta(n)$ with the normal (see the figure). For $n = \sqrt{3}$ the value of θ is 60° and $\frac{d\theta}{dn} = m$. The value of m is **[JEE Adv. 2015]**



4 Fill in the Blanks

11. A ray of light is incident normally on one of the faces of a prism of apex angle 30° and refractive index $\sqrt{2}$. The angle of deviation of the ray is... degrees. **[1997 - 2 Marks]**

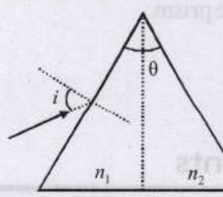
12. A ray of light undergoes deviation of 30° when incident on an equilateral prism of refractive index $\sqrt{2}$. The angle made by the ray inside the prism with the base of the prism is.................. **[1992 - 1 Mark]**

5 True / False

13. A beam of white light passing through a hollow prism give no spectrum. **[1983 - 2 Marks]**

6 MCQs with One or More than One Correct Answer

14. For a prism of prism angle $\theta = 60°$, the refractive indices of the left half and the right half are, respectively, n_1 and n_2 ($n_2 \geq n_1$) as shown in the figure. The angle of incidence i is chosen such that the incident light rays will have minimum deviation if $n_1 = n_2 = n = 1.5$. For the case of unequal refractive indices $n_1 = n$ and $n_2 = n + \Delta n$ (where $\Delta n << n$), the angle of emergence $e = i + \Delta e$. Which of the following statement(s) is (are) correct? **[Adv. 2021]**



(a) The value of Δe (in radians) is greater than that of Δn
(b) Δe is proportional to Δn
(c) Δe lies between 2.0 and 3.0 milliradians, if $\Delta n = 2.8 \times 10^{-3}$
(d) Δe lies between 1.0 and 1.6 milliradians, if $\Delta n = 2.8 \times 10^{-3}$

15. For an isosceles prism of angle A and refractive index μ, it is found that the angle of minimum deviation $\delta_m = A$. **[Adv. 2017]**

Which of the following options is/are correct?

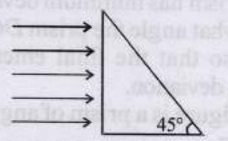
(a) For the angle of incidence $i_1 = A$, the ray inside the prism is parallel to the base of the prism
(b) For this prism, the refractive index μ and the angle of prism A are related as $A = \frac{1}{2}\cos^{-1}\left(\frac{\mu}{2}\right)$
(c) At minimum deviation, the incident angle i_1 and the refracting angle r_1 at the first refracting surface are related by $r_1 = (i_1/2)$
(d) For this prism, the emergent ray at the second surface will be tangential to the surface when the angle of incidence at the first surface is

$$i_1 = \sin^{-1}\left[\sin A\sqrt{4\cos^2\frac{A}{2} - 1} - \cos A\right]$$

16. A thin prism P_1 with angle 4° and made from glass of refractive index 1.54 is combined with another thin prism P_2 made from glass of refractive index 1.72 to produce dispersion without deviation. The angle of the prism P_2 is **[1990 - 2 Marks]**

(a) 5.33° (b) 4° (c) 3° (d) 2.6°

17. A beam of light consisting of red, green and blue colours is incident on a right angled prism, fig. The refractive indices of the material of the prism for the above red, green and blue wavelengths are 1.39, 1.44 and 1.47 respectively. The prism will **[1989 - 2 Mark]**

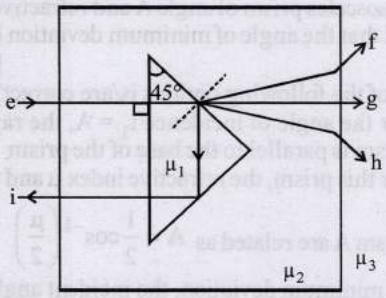


(a) separate part of the red colour from the green and blue colours
(b) separate part of the blue colour from the red and green colours
(c) separate all the three colours from one another
(d) not separate even partially any colour from the other two colours.

7 Match the Following

18. A right angled prism of refractive index μ_1 is placed in a rectangular block of refractive index μ_2, which is surrounded by a medium of refractive index μ_3, as shown in the figure. A ray of light 'e' enters the rectangular block at normal incidence. Depending upon the relationships between μ_1, μ_2 and μ_3, it takes one of the four possible paths 'ef', 'eg', 'eh' or 'ei'. Match the paths in List I with conditions of refractive indices in List II and select the correct answer using the codes given below the lists: **[Adv. 2013]**

List I		List II	
P.	$e \to f$	1.	$\mu_1 > \sqrt{2}\mu_2$
Q.	$e \to g$	2.	$\mu_2 > \mu_1$ and $\mu_2 > \mu_3$
R.	$e \to h$	3.	$\mu_1 = \mu_2$
S.	$e \to i$	4.	$\mu_2 < \mu_1 < \sqrt{2}\mu_2$ and $\mu_2 > \mu_3$

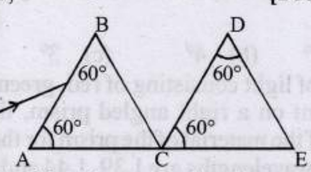


Codes:

	P	Q	R	S
(a)	2	3	1	4
(b)	1	2	4	3
(c)	4	1	2	3
(d)	2	3	4	1

10 Subjective Problems

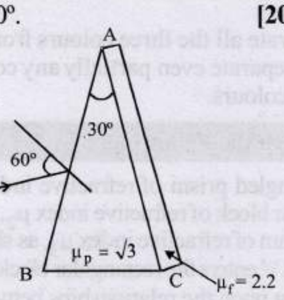
19. Two identical prisms of refractive index $\sqrt{3}$ are kept as shown in the figure. A light ray strikes the first prism at face *AB*. Find, **[2005 - 4 Marks]**



(a) the angle of incidence, so that the emergent ray from the first prism has minimum deviation.

(b) through what angle the prism *DCE* should be rotated about *C* so that the final emergent ray also has minimum deviation.

20. Shown in the figure is a prism of angle 30° and refractive index $\mu_p = \sqrt{3}$. Face AC of the prism is covered with a thin film of refractive index $\mu_f = 2.2$. A monochromatic light of wavelength $\lambda = 550$ nm fall on the face *AB* at an angle of incidence of 60°. **[2003 - 4 Marks]**



Calculate

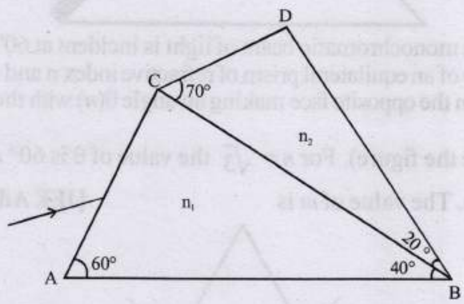
(a) angle of emergence.

(b) minimum value of thickness *t* so that intensity of emergent ray is maximum.

21. A prism of refractive index n_1 and another prism of refractive index n_2 are stuck together without a gap as shown in Figure. The angles of the prisms are as shown. n_1 and n_2 depend on λ, the wavelength of light, according to

$$n_1 = 1.20 + \frac{10.8 \times 10^4}{\lambda^2} \text{ and } n_2 = 1.45 + \frac{1.80 \times 10^4}{\lambda^2} \text{ where}$$

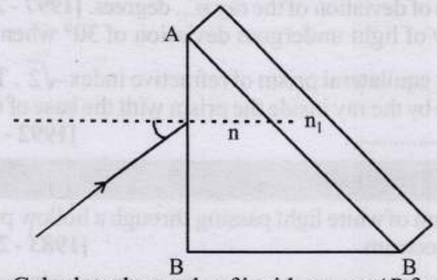
λ is in *nm*. **[1998 - 8 Marks Similar July 27, 2021 (I)]**



(a) Calculate the wavelength λ_0 for which rays incident at any angle on the interface *BC* pass through without bending at that interface.

(b) For light of wavelength λ_0, find the angle of incidence i on the face *AC* such that the deviation produced by the combination of prisms is minimum.

22. A right angled prism (45°–90°–45°) of refractive index n has a plate of refractive index $n_1 (n_1 < n)$ cemented to its diagonal face. The assembly is in air. A ray is incident on *AB*.



(i) Calculate the angle of incidence at *AB* for which the ray strikes the diagonal face at the critical angle.

(ii) Assuming $n = 1.352$ calculate the angle of incidence at *AB* for which the refracted ray passes through the diagonal face undeviated. **[1996 - 3 Marks]**

23. A ray of light is incident at an angle of 60° on one face of prism which has an angle of 30°. The ray emerging out of the prism makes an angle of 30° with the incident ray. Show that the emergent ray is perpendicular to the face through which it emerges and calculate the refractive index of the material of the prism. **[1978]**

Topic-5: Optical Instruments

1 MCQs with One Correct Answer

1. In a compound microscope, the intermediate image is **[2000S]**

(a) virtual, erect and magnified

(b) real, erect and magnified

(c) real, inverted and magnified

(d) virtual, erect and reduced

2. The focal lengths of the objective and the eye piece of a compound microscope are 2.0 cm and 3.0 cm, respectively. The distance between the objective and the eye piece is 15.0 cm. The final image formed by the eye piece is at infinity. The two lenses are thin. The distance in cm of the object and the image produced by the objective, measured from the objective lens, are respectively **[1995S]**
(a) 2.4 and 12.0 (b) 2.4 and 15.0
(c) 2.0 and 12.0 (d) 2.0 and 3.0

4 Fill in the Blanks

3. The resolving power of electron microscope is higher that that of an optical microscope because the wavelength of electrons is .... than the wavelength of visible light. **[1992 - 1 Mark]**

6 MCQs with One or More than One Correct Answer

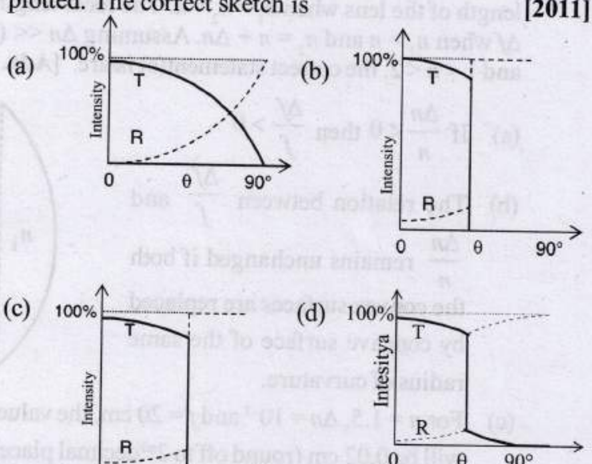
4. A planet is observed by an astronomical refracting telescope having an objective of focal length 16 m and an eyepiece of focal length 2 cm. **[1992 - 2 Marks]**
(a) The distance between the objective and the eyepiece is 16.02 m
(b) The angular magnification of the planet is – 800
(c) The image of the planet is inverted
(d) The objective is larger then the eyepiece

5. An astronomical telescope has an angular magnification of magnitude 5 for distant objects. The separation between the objective and the eyepiece is 36 cm and the final image is formed at infinity. The focal length f_0 of the objective and the focal length f_e of the eyepiece are **[1989 - 2 Marks]**
(a) $f_0 = 45$ cm and $f_e = -9$ cm
(b) $f_0 = 50$ cm and $f_e = 10$ cm
(c) $f_0 = 7.2$ cm and $f_e = 5$ cm
(d) $f_0 = 30$ cm and $f_e = 6$ cm.

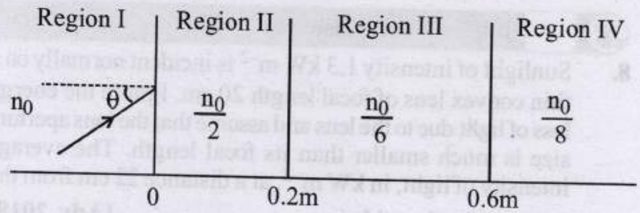
Topic-6: Miscellaneous (Mixed Concepts) Problems

1 MCQs with One Correct Answer

1. A light ray travelling in glass medium is incident on glass-air interface at an angle of incidence θ. The reflected (R) and transmitted (T) intensities, both as function of θ, are plotted. The correct sketch is **[2011]**

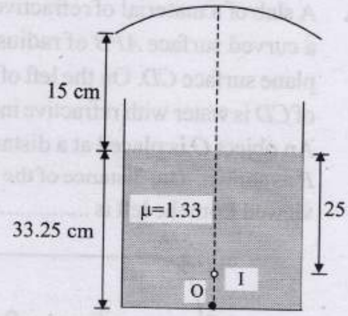


2. A light beam is travelling from Region I to IV (figure). The refractive index in regionals I, II, III and IV are $n_0, \frac{n_0}{2}, \frac{n_0}{6}$ and $\frac{n_0}{8}$ respectively. The angle of incidence θ for which the beam just misses entering region IV is – **[2008]**



(a) $\sin^{-1}(3/4)$ (b) $\sin^{-1}(1/8)$
(c) $\sin^{-1}(1/4)$ (d) $\sin^{-1}(1/3)$

3. A container is filled with water (μ = 1.33) upto a height of 33.25 cm. A concave mirror is placed 15 cm above the water level and the image of an object placed at the bottom is formed 25 cm below the water level. Focal length of the mirror is **[2005S]**
(a) 15 cm
(b) 20 cm
(c) –18.31 cm
(d) 10 cm

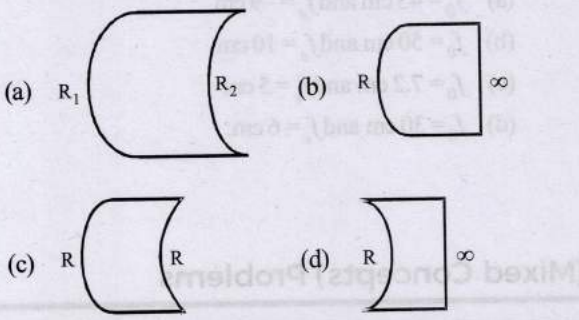


4. A point object is placed at the centre of a glass sphere of radius 6 cm and refractive index 1.5. The distance of virtual image from the surface is **[2004S]**
(a) 6 cm (b) 4 cm (c) 12 cm (d) 9 cm

5. A source emits sound of frequency 600 Hz inside water. The frequency heard in air will be equal to (velocity of sound in water = 1500 m/s, velocity of sound in air = 300 m/s) **[2004S]**
(a) 3000 Hz (b) 120 Hz (c) 600 Hz (d) 6000 Hz

6. A beam of white light is incident on glass air interface from glass to air such that green light just suffers total internal reflection. The colors of the light which will come out to air are **[2004S]**
(a) Violet, Indigo, Blue (b) All colors except green
(c) Yellow, Orange, Red (d) White light

7. Which one of the following spherical lenses does not exhibit dispersion? The radii of curvature of the surfaces of the lenses are as given in the diagrams. **[2002S]**



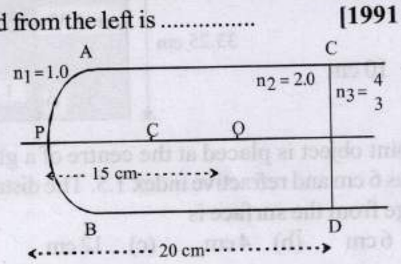
2 Integer Value Answer

8. Sunlight of intensity 1.3 kW m^{-2} is incident normally on a thin convex lens of focal length 20 cm. Ignore the energy loss of light due to the lens and assume that the lens aperture size is much smaller than its focal length. The average intensity of light, in kW m^{-2}, at a distance 22 cm from the lens on the other side is ________. **[Adv. 2018]**

4 Fill in the Blanks

9. If ε_0 and μ_0 are, respectively, the electric permittivity and magnetic permeability of free space, ε and μ the corresponding quantities in a medium, the index of refraction of the medium in terms of the above parameters is .... **[1992 - 1 Mark]**

10. A slab of a material of refractive index 2 shown in fig. has a curved surface *APB* of radius of curvature 10 cm and a plane surface *CD*. On the left of *APB* is air and on the right of *CD* is water with refractive indices as given in the figure. An object *O* is placed at a distance of 15 cm from the pole *P* as shown. The distance of the final image of *O* from *P*, as viewed from the left is .... **[1991 - 2 Marks]**



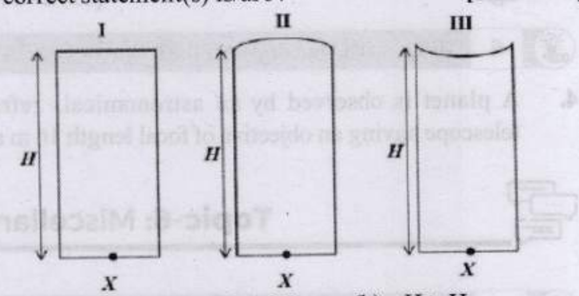
5 True / False

11. The intensity of light at a distance '*r*' from the axis of a long cylindrical source is inversely proportional to '*r*'. **[1981 - 2 Marks]**

12. The setting sun appears higher in the sky than it really is. **[1980]**

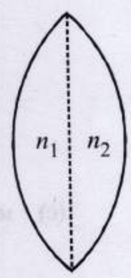
6 MCQs with One or More than One Correct Answer

13. Three glass cylinders of equal height H = 30 cm and same refractive index $n = 1.5$ are placed on a horizontal surface as shown in figure. Cylinder I has a flat top, cylinder II has a convex top and cylinder III has a concave top. The radii of curvature of the two curved tops are same (R = 3 m), If H_1, H_2, and H_3 are the apparent depths of a point *X* on the bottom of the three cylinders, respectively, the correct statement(s) is/are : **[Adv. 2019]**



(a) $0.8\text{ cm} < (H_2 - H_1) < 0.9\text{ cm}$ (b) $H_2 > H_1$
(c) $H_3 > H_1$ (d) $H_2 > H_3$

14. A thin convex lens is made of two materials with refractive indices n_1 and n_2, as shown in figure. The radius of curvature of the left and right spherical surface are equal. *f* is the focal length of the lens when $n_1 = n_2 = n$. The focal length is $f + \Delta f$ when $n_1 = n$ and $n_2 = n + \Delta n$. Assuming $\Delta n << (n-1)$ and $1 < n < 2$, the correct statement(s) is/are. **[Adv. 2019]**



(a) If $\frac{\Delta n}{n} < 0$ then $\frac{\Delta f}{f} > 0$

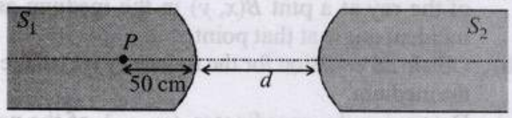
(b) The relation between $\frac{\Delta f}{f}$ and $\frac{\Delta n}{n}$ remains unchanged if both the convex surfaces are replaced by concave surface of the same radius of curvature.

(c) For $n = 1.5$, $\Delta n = 10^{-3}$ and $f = 20$ cm, the value of $|\Delta f|$ will be 0.02 cm (round off to 2[nd] decimal place).

(d) $\left|\frac{\Delta f}{f}\right| < \left|\frac{\Delta n}{n}\right|$

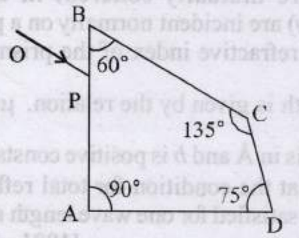
15. Two identical glass rods S_1 and S_2 (refractive index = 1.5) have one convex end of radius of curvature 10 cm. They are placed with the curved surfaces at a distance *d* as shown in the figure, with their axes (shown by the dashed line) aligned. When a point source of light *P* is placed inside rod S_1 on its axis at a distance of 50 cm from the curved face,

the light rays emanating from it are found to be parallel to the axis inside S_2. The distance d is **[Adv. 2015]**



(a) 60 cm (b) 70 cm (c) 80 cm (d) 90 cm

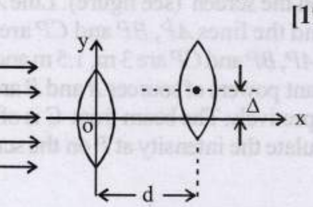
16. A ray OP of monochromatic light is incident on the face AB of prism ABCD near vertex B at an incident angle of 60° (see figure). If the refractive index of the material of the prism is $\sqrt{3}$, which of the following is (are) correct? **[2010]**



(a) The ray gets totally internally reflected at face CD
(b) The ray comes out through face AD
(c) The angle between the incident ray and the emergent ray is 90°
(d) The angle between the incident ray and the emergent ray is 120°

17. A concave mirror is placed on a horizontal table, with its axis directed vertically upwards. Let *O* be the pole of the mirror and C its centre of curvature. A point object is placed at *C*. It has a real image, also located at *C*. If the mirror is now filled with water, the image will be. **[1998 - 2 Marks]**
(a) real, and will remain at *C*.
(b) real, and located at a point between *C* and ∞.
(c) virtual, and located at a point between *C* and *O*.
(d) real, and located at a point between *C* and *O*

18. Two thin convex lenses of focal lengths f_1 and f_2 are separated by a horizontal distance d (where $d < f_1$, $d < f_2$) and their centres are displaced by a vertical separation Δ as shown in the fig. **[1993-2 Marks]**



Taking the origin of coordinates *O*, at the centre of the first lens the x and y coordinates of the focal point of this lens system, for a parallel beam of rays coming from the left, are given by:

(a) $x=\dfrac{f_1f_2}{f_1+f_2}, y=\Delta$

(b) $x=\dfrac{f_1(f_2+d)}{f_1+f_2-d}, y=\dfrac{\Delta}{f_1+f_2}$

(c) $x=\dfrac{f_1f_2+d(f_1-d)}{f_1+f_2-d}, y=\dfrac{\Delta(f_1-d)}{f_1+f_2-d}$

(d) $x=\dfrac{f_1f_2+d(f_1-d)}{f_1+f_2-d}, y=0$

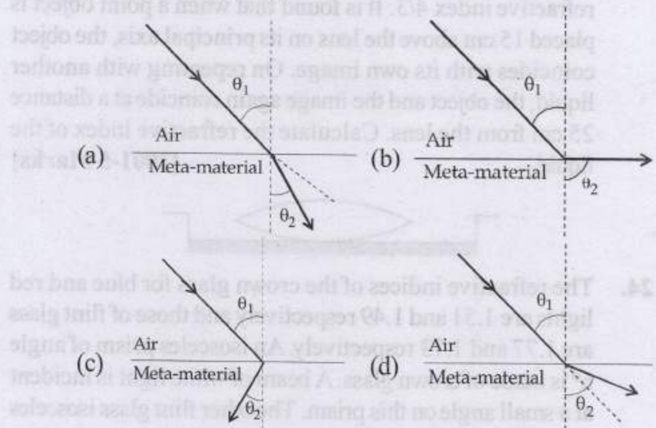
8 Comprehension/Passage Based Questions

Passage

Most materials have the refractive index, $n > 1$. So, when a light ray from air enters a naturally occurring material, then by Snell's law, $\dfrac{\sin\theta_1}{\sin\theta_2}=\dfrac{n_2}{n_1}$, it is understood that the refracted ray bends towards the normal. But it never emerges on the same side of the normal as the incident ray. According to electromagnetism, the refractive index of the medium is given by the relation, $n = c/v = \pm\sqrt{\varepsilon_r\mu_r}$, where *c* is the speed of electromagnetic waves in vacuum, *v* its speed in the medium, ε_r and μ_r are the relative permittivity and permeability of the medium respectively.

In normal materials, both ε_r and μ_r, are positive, implying positive *n* for the medium. When both ε_r and μ_r are negative, one must choose the negative root of *n*. Such negative refractive index materials can now be artificially prepared and are called meta-materials. They exhibit significantly different optical behavior, without violating any physical laws. Since *n* is negative, it results in a change in the direction of propagation of the refracted light. However, similar to normal materials, the frequency of light remains unchanged upon refraction even in meta-materials. **[2012]**

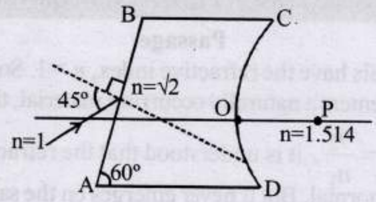
19. For light incident from air on a meta-material, the appropriate ray diagram is



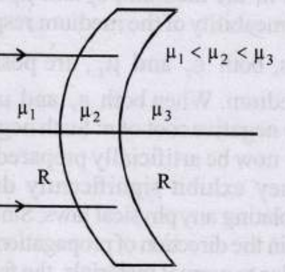
20. Choose the correct statement.
(a) The speed of light in the meta-material is $v = c|n|$
(b) The speed of light in the meta-material is $v=\dfrac{c}{|n|}$
(c) The speed of light in the meta-material is $v = c$.
(d) The wavelength of the light in the meta-material (λ_m) is given by $\lambda_m = \lambda_{air}|n|$, where λ_{air} is wavelength of the light in air.

10 Subjective Problems

21. A ray is incident on a medium consisting of two boundaries, one plane and other curved as shown in the figure. The plane surface makes an angle 60° with horizontal and curved surface has radius of curvature 0.4 m. The refractive indices of the medium and its environment are shown in the figure. If after refraction at both the surfaces the ray meets principle axis at *P*, find *OP*. **[2004 - 2 Marks]**



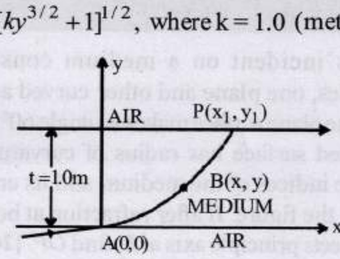
22. Find the focal length of the lens shown in the figure. The radii of curvature of both the surfaces are equal to *R*. **[2003-2 Marks]**



23. A thin biconvex lens of refractive index 3/2 is placed on a horizontal plane mirror as shown in the figure. The space between the lens and the mirror is then filled with water of refractive index 4/3. It is found that when a point object is placed 15 cm above the lens on its principal axis, the object coincides with its own image. On repeating with another liquid, the object and the image again coincide at a distance 25 cm from the lens. Calculate the refractive index of the liquid. **[2001-5 Marks]**

24. The refractive indices of the crown glass for blue and red lights are 1.51 and 1.49 respectively and those of flint glass are 1.77 and 1.73 respectively. An isosceles prism of angle 6° is made of crown glass. A beam of white light is incident at a small angle on this prism. The other flint glass isosceles prism is combined with the crown glass prism such that there is no deviation of the incident light. (a) Determine the angle of the flint glass prism. (b) Calculate the net dispersion of the combined system. **[2001 - 5 Marks]**

25. A ray of light travelling in air is incident at grazing angle (incident angle $\simeq 90°$) on a long rectangular slab of a transparent medium of thickness t = 1.0 m (see figure below). The point of incidence is the origin *A*(0, 0). The medium has a variable index of refraction *n*(*y*) given by $n(y) = [ky^{3/2} + 1]^{1/2}$, where k = 1.0 (metre)$^{-3/2}$



The refractive index of air is 1.0. **[1995 - 10 Marks]**

(a) Obtain a relation between the slope of the trajectory of the ray at a pint *B*(*x*, *y*) in the medium and the incident angle at that point.

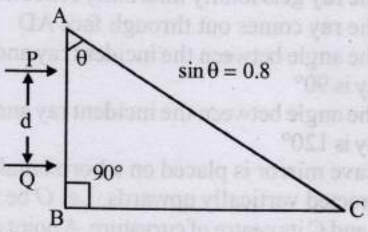
(b) Obtain an equation for the trajectory *y*(*x*) of the ray in the medium.

(c) Determine the coordinates (x_1, y_1) of the point *P*, where the ray intersects the upper surface of the slab-air boundary.

(d) Indicate the path of the ray subsequently.

26. Two parallel beams of light *P* and *Q* (separation d) containing radiations of wavelengths 4000 Å and 5000 Å (which are mutually coherent in each wavelength separately) are incident normally on a prism as shown in fig. The refractive index of the prism as a function of wavelength is given by the relation. $\mu(\lambda) = 1.20 + \frac{b}{\lambda^2}$

where λ is in Å and *b* is positive constant. The value of *b* is such that the condition for total reflection of the face *AC* is just satisfied for one wave length and is not satisfied for the other. **[1991 - 2 + 2 + 4 Marks]**

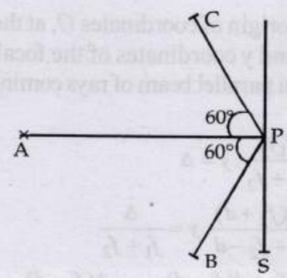


(a) Find the value of *b*.

(b) Find the deviation of the beams transmitted through the face *AC*

(c) A convergent lens is used to bring these transmitted beams into focus. If the intensities of transmission form the face *AC*, are 41 and *I* respectively, find the resultant intensity at the focus.

27. Screen S is illuminated by two point sources *A* and *B*. Another source *C* sends a parallel beam of light towards point *P* on the screen (see figure). Line *AP* is normal to the screen and the lines *AP*, *BP* and *CP* are in one plane. The distance *AP*, *BP* and *CP* are 3 m, 1.5 m and 1.5 m respectively. The radiant powers of sources *A* and *B* are 90 watts and 180 watts respectively. The beam from C is of intensity 20 watts/m^2. Calculate the intensity at *P* on the screen. **[1982 - 5 Marks]**



28. An object is placed 21 cm in front of a concave mirror of radius of curvature 10 cm. A glass slab of thickness 3 cm and refractive index 1.5 is then placed close to the mirror in the space between the object and the mirror.
Find the position of the final image formed. (You may take the distance of the near surface of the slab from the mirror to be 1 cm. **[1980]**

29. A rectangular block of glass is placed on a printed page lying on a horizontal surface. Find the minimum value of the refractive index of glass for which the letters on the page are not visible from any of the vertical faces of the block. **[1979]**

Answer Key

Topic-1 : Plane Mirror, Spherical Mirror and Reflection of Light

1. (a) **2.** (b) **3.** (d) **4.** (d) **5.** (40) **6.** (3) **7.** (3) **9.** (a, b) **10.** (d) **11.** (c, d) **12.** (c)

Topic-2 : Refraction of Light at Plane Surface and Total Internal Reflection

1. (c) **2.** (c) **3.** (c) **4.** (a) **5.** (b) **6.** (b) **7.** (b) **8.** (d) **9.** (a) **10.** (b) **11.** (a, b) **12.** (a) **13.** (4) **14.** (8) **15.** (2) **16.** (6) **17.** (50) **22.** (a, c, d) **23.** (c, d) **24.** (a, c) **25.** (d)

Topic-3 : Refraction at Curved Surface Lenses and Power of Lens

1. (b) **2.** (c) **3.** (c) **4.** (b) **5.** (b) **6.** (b) **7.** (c) **8.** (b) **9.** (a) **10.** (b) **11.** (d) **12.** (a) **13.** (c) **14.** (c) **15.** (80 or 150 or 220) **16.** (7) **17.** (6) **18.** (6) **22.** True **23.** False **24.** (a, b) **25.** (a, d) **26.** (a, d) **27.** (a) **28.** (b) **29.** (b, c) **30.** (b, d) **31.** (a) **32.** (a) **33.** (a) **34.** (b)

Topic-4 : Prism and Dispersion of Light

1. (a) **2.** (c) **3.** (a) **4.** (b) **5.** (c) **6.** (c) **7.** (a) **8.** (12) **9.** (1.50) **10.** (2) **13.** True **14.** (b, c) **15.** (a, c, d) **16.** (c) **17.** (a) **18.** (d)

Topic-5 : Optical Instruments

1. (c) **2.** (a) **4.** (a, b, c, d) **5** (d)

Topic-6 : Miscellaneous (Mixed Concepts) Problems

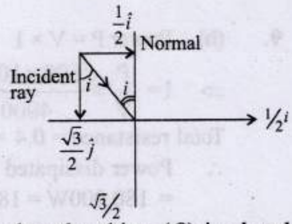
1. (c) **2.** (b) **3.** (c) **4.** (a) **5.** (c) **6.** (c) **7.** (c) **8.** (130) **11.** True **12.** True **13.** (b,d) **14.** (a, b, c) **15.** (b) **16.** (a, b, c) **17.** (d) **18.** (c) **19.** (c) **20.** (b)

Hints & Solutions

Topic-1: Plane Mirror, Spherical Mirror and Reflection of Light

1. **(a)**
From figure, tan

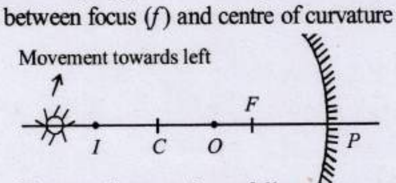
$i = \dfrac{1/2}{\sqrt{3}/2} = \dfrac{1}{\sqrt{3}}$



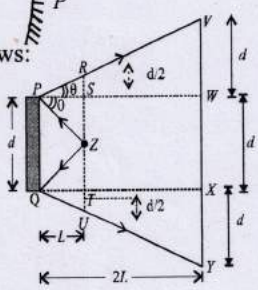
$\therefore\ i = 30°$

(Angle of incidence)

2. **(b)** As shown in the figure, when the object (*O*) is placed between *F* and *C*, the image (*I*) is formed beyond *C*. It is in this condition that when the student shifts his eyes towards left, the image appears to the right of the object pin. Object O lies between focus (*f*) and centre of curvature (2*f*) $f < x < 2f$.



3. **(d)** The ray diagram is as follows:
From figure, $WX = PQ = d$



$ZS = RS = \dfrac{d}{2}$

$\because\ PW = 2PS$

$\therefore\ VW = 2RS = 2\dfrac{d}{2} = d$

Similarly, $XY = d$

$\therefore\ VY = VW + WX + XY = d + d + d = 3d$

4. **(d)** From mirror formula, $\dfrac{1}{v} + \dfrac{1}{u} = \dfrac{1}{f}\ \therefore\ -\dfrac{dv}{v^2} - \dfrac{du}{u^2} = 0$

$\therefore\ \dfrac{dv}{du} = \dfrac{-v^2}{u^2} = -\left(\dfrac{f}{u-f}\right)^2 \Rightarrow dv = -\left(\dfrac{f}{4-f}\right)^2 du$

$\therefore\ \text{Image size} = \left(\dfrac{f}{u-f}\right)^2 \times b$

5. **(40)** From figure ΔABE and CDE are similar triangle so,

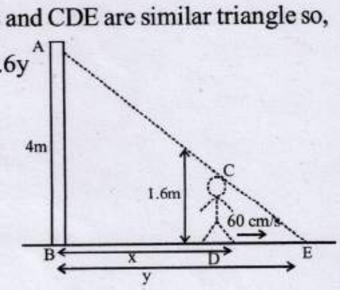
$\dfrac{4}{y} = \dfrac{1.6}{y-x} \Rightarrow 4y - 4x = 1.6y$

$\Rightarrow 2.4y = 4x$

$\therefore\ x = 0.6y$

Differentiating both sides

$\dfrac{dx}{dt} = 0.6 \times \dfrac{dy}{dt}$



$\left(\dfrac{dx}{dt} = \text{speed of person} = 60\text{cm/s and } \dfrac{dy}{dt} = \text{speed of tip of person's shadow}\right)$

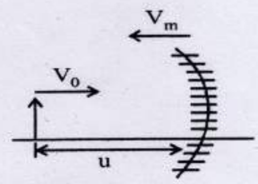
$60 = 0.6 \times \dfrac{dy}{dt}\qquad \therefore\ \dfrac{dy}{dt} = 100\text{cm/s}$

Speed of tip of person's shadow on the ground, w.r.t person $= 100 - 60 = 40$ cm/s

6. **(3)** We have, $u = -30$cm, $f = -10$ cm

$\dfrac{1}{v} = \dfrac{1}{f} - \dfrac{1}{u} = \dfrac{1}{-10} - \dfrac{1}{-30} = \dfrac{1}{30} - \dfrac{1}{10} = \dfrac{1-3}{30} = \dfrac{-1}{15}$

So, $v = -15$ cm



As, $\vec{V}_{I,M} = -\left(\dfrac{v}{u}\right)^2 \vec{V}_{O,M}$

$\Rightarrow \vec{V}_I - \vec{V}_M = -\left(\dfrac{v}{u}\right)^2 \left(\vec{V}_0 - \vec{V}_m\right)$

$\Rightarrow 0 - \vec{V}_m = -\dfrac{1}{4}\left(15\hat{i} - \vec{V}_m\right)$

$\Rightarrow -\vec{V}_m = -\dfrac{15}{4}\hat{i} + \dfrac{1}{4}\vec{V}_m \Rightarrow \dfrac{-5}{4}\vec{V}_m = \dfrac{-15}{4}\hat{i} \Rightarrow \vec{V}_m = 3\hat{i}$

7. **(3)** **Using mirror formula for first position**

$u_1 = ?, v_1 = \dfrac{25}{3}\text{cm}, f = +10\text{cm}\left(=\dfrac{R}{2}\right)$

$\dfrac{1}{v_1} + \dfrac{1}{u_1} = \dfrac{1}{f}, \dfrac{3}{25} + \dfrac{1}{u_1} = \dfrac{1}{10}\ \therefore u_1 = -50\text{ m}$

Using mirror formula for the second position

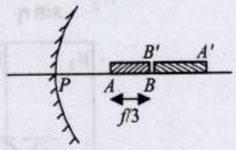
$u_2 = ?, v_2 = \dfrac{50}{7}$ and $f = 10$ cm

$\dfrac{1}{v_2} + \dfrac{1}{u_2} = \dfrac{1}{f} \Rightarrow \dfrac{7}{50} + \dfrac{1}{u_2} = \dfrac{1}{10} \Rightarrow \dfrac{1}{u_2} = \dfrac{1}{10} - \dfrac{7}{50}$

$u_2 = -25$m

Speed of object $= \dfrac{u_1 - u_2}{\text{time}} = \dfrac{25}{30} \times \dfrac{18}{5} = 3\,\text{km h}^{-1}$

8. Since the image formed is real and elongated, the object lies between focus F and centre of curvature C the situation is as shown in the figure. Since the image of B is formed at B' itself



$\therefore$ B is situated at the centre of curvature that is at a distance of $2f$ from the pole.

$\therefore \quad PA = 2f - \frac{f}{3} = \frac{5f}{3}$

Let us find the image of A. For point A, $u = -\frac{5f}{3}, v = ?$

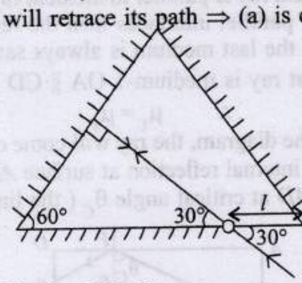
From formula, $\frac{1}{u} + \frac{1}{v} = \frac{1}{f} \Rightarrow \frac{1}{\frac{-5f}{3}} + \frac{1}{v} = \frac{1}{-f}$

$\Rightarrow \frac{1}{v} = -\frac{1}{f} + \frac{3}{5f} \quad \therefore v = -2.5f$

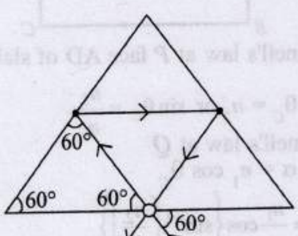
Image length of rod $= 2.5f - 2f = 0.5f$

$\therefore$ Magnification $M = \frac{v}{u} = \frac{0.5f}{f/3} = 1.5 \left[\text{Given } u = \frac{f}{3} \right]$

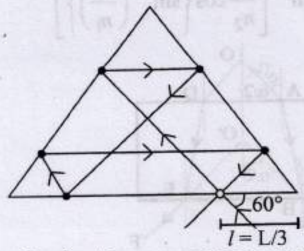
9. **(a, b)** As we can see, for $\theta = 30°$ the ray will incident normally and hence will retrace its path $\Rightarrow$ (a) is correct.



For $\theta = 60°$, $\ell = L/2$. Then, we get ray diagram shown below. Clearly ray of light comes after two reflections $\Rightarrow$ (b) is correct.

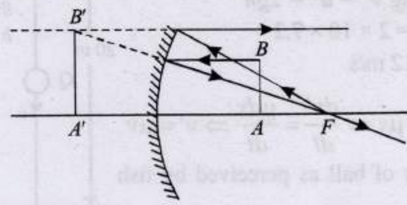


If $\theta = 60°$ and $\ell = L/3$, then we get ray diagram as shown below



Clearly, after 5 reflections, ray comes out. So (c) and (d) are incorrect.

10. **(d)** Distance of point A from the mirror is $f/2$.



From mirror formula,

$\frac{1}{v} + \frac{1}{f/2} = \frac{1}{-f} \Rightarrow \frac{1}{v} = \frac{2}{f} - \frac{1}{f} = \frac{1}{f} \quad \therefore v = f$

Image $A'B'$ of line AB should be I principle axis. Image of F will be formed at infinity.

Also light ray from infinity or towards infinity seems parallel to the principle axis of the mirror.

11. **(c, d)** Given $f = -24$ cm

Applying mirror formula, $\frac{1}{v} + \frac{1}{u} = \frac{1}{f}$

$u - v$ values of options (a) and (b) match with mirror formula. Whereas option (c) and (d) do not match with mirror formula.

For (66, 33)

$\Rightarrow \frac{1}{v} = \frac{1}{f} - \frac{1}{u} = \frac{1}{-24} + \frac{1}{66} = \frac{-66+24}{24 \times 66} = \frac{-42}{24 \times 66}$

$\Rightarrow v = -\frac{24 \times 66}{42} = -37.7$

But the value of $v = 33$. The absolute error is $37.7 - 33 = 4.7$ cm which is greater than 0.2 cm. Therefore a wrong reading.

For (78, 39) when $u = 78$ then

$\frac{1}{v} + \frac{1}{-78} = \frac{1}{-24} \Rightarrow v = -34.67$

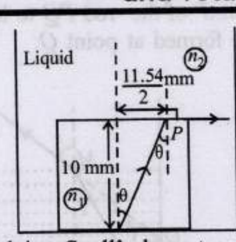
The absoluate error is 39 – 34.67 = 4.33 which is greater than 0.2 cm.

12. **(c)** The formula connecting u, v and f for a spherical mirror $\frac{1}{u} + \frac{1}{v} = \frac{1}{f}$ is valid only for mirrors of small apertures where the size of aperture is very small as compared to the radius of curvature of the mirror.

Laws of reflection are valid for plane as well as large spherical surfaces. The laws of reflection are valid when ever the light is reflected.

Topic-2: Refraction of Light at Plane Surface and Total Internal Reflection

1. **(c)**



Applying Snell's law at point P

$n_1 \sin\theta = n_2 \sin 90°$

$\sin\theta = \frac{11.54/2}{\sqrt{10^2 + (11.54/2)^2}}$ and n_2 = refractive index of liquid

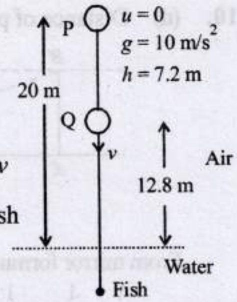
$\therefore \quad n_2 = 2.72 \times \frac{11.54/2}{\sqrt{(10)^2 + \left(\frac{11.54}{2}\right)^2}} \quad \therefore n_2 = 1.36$

2. **(c)** Ball falling *P* to *Q*

Applying $v^2 - u^2 = 2gh$

$v^2 - 0^2 = 2 \times 10 \times 7.2$

$\Rightarrow v = 12$ m/s

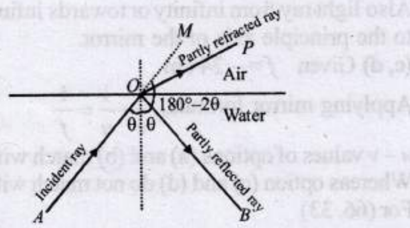


as $x' = \mu x \Rightarrow \frac{dx'}{dt} = \frac{\mu dx}{dt} \Rightarrow v' = \mu v$

Velocity of ball as perceived by fish

$v' = {}_{w}\mu \times v = \frac{4}{3} \times 12 = 16$ m/s

3. **(c)** The incident ray is partly reflected and partly refracted. $\angle MOB = 180° - 2\theta > \angle POB$ the angle between refracted and reflected ray.



4. **(a)** Distance of virtual image from surface = 6 cm

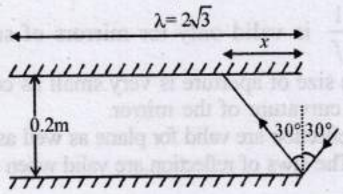
The rays coming from the point object fall on the glass-air interface normally and hence pass undeviated. Therefore if we retrace the path of the refracted rays backwards, the image will be formed at the centre only.

5. **(b)** From snell's law, $\mu_g \sin i = \mu_{air} \sin 90°$

$\therefore \quad \mu_g = \frac{1}{\sin i}$

6. **(b)** Maximum number of reflection $= \frac{2\sqrt{3}}{x}$

where $x = 0.2 \tan 30° = 0.2/\sqrt{3}$.



$\therefore$ Maximum number of reflection $= \frac{2\sqrt{3}}{0.2/\sqrt{3}} = 30$

7. **(b)** For the image of point *P* of the rod *PQ* to be seen by the observer, it should be formed at point *Q*.

In ΔQNS,

$NS = QS = 2h$

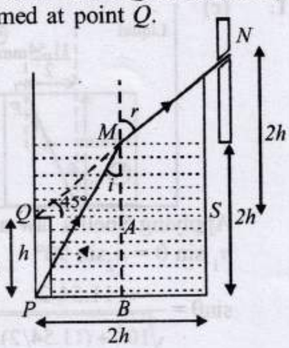
$\therefore \quad \angle NQS = 45°$

$\therefore \quad r = 45°$

Now in ΔQMA,

$\angle MQA = 45°$

$\therefore \quad MA = QA = h$



${}_{2}^{1}\mu = \frac{\sin r}{\sin i} = \frac{\sin 45°}{\sin i}$ ...(i)

In ΔPMB,

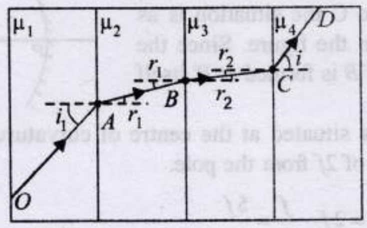
$PM^2 = 4h^2 + h^2 = 5h^2$

$\therefore \quad \sin i = \frac{h}{\sqrt{5}h} = \frac{1}{\sqrt{5}}$ ...(ii)

From eq. (i) and (ii) $\Rightarrow {}_{2}^{1}\mu = \frac{\sin 45°}{\sin i} = \sqrt{\frac{5}{2}}$

8. **(d)** Applying Snell's law at *A*,

$${}^{1}\mu_2 = \frac{\sin i}{\sin r_1} = \frac{\mu_2}{\mu_1} \quad \text{...(i)}$$



Applying Snell's law at *B*,

$${}^{2}\mu_3 = \frac{\sin r_1}{\sin r_2} = \frac{\mu_3}{\mu_2} \quad \text{...(ii)}$$

Again applying Snell's law at *C*

$${}^{3}\mu_4 = \frac{\sin r_2}{\sin i} = \frac{\mu_4}{\mu_3} \quad \text{...(iii)}$$

Multiplying eq. (i), (ii) and (iii)

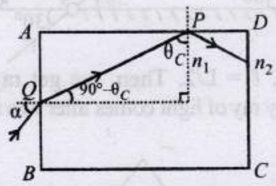
$$\frac{\mu_2}{\mu_1} \times \frac{\mu_3}{\mu_2} \times \frac{\mu_4}{\mu_3} = \frac{\sin i}{\sin r_1} \times \frac{\sin r_1}{\sin r_2} \times \frac{\sin r_2}{\sin i}$$

$\therefore \quad \frac{\mu_4}{\mu_1} = 1 \quad \therefore \quad \mu_1 = \mu_4$

If the emergent ray is parallel to incident ray after travelling a number of parallel interfaces then the refractive index of the first and the last medium is always same.

Here incident ray is medium-1 OA || CD emergent ray in medium-4 $\quad \therefore \quad \mu_1 = \mu_4$

9. **(a)** From the diagram, the ray will come out from *CD* if it suffers total internal reflection at surface *AD*, i.e., it strikes the surface *AD* at critical angle θ_C (the limiting case).



Applying Snell's law at *P* face AD of slab

$n_1 \sin \theta_C = n_2$ or $\sin \theta_C = \frac{n_2}{n_1}$

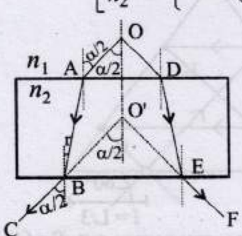
Applying Snell's law at *Q*

$n_2 \sin \alpha = n_1 \cos \theta_C$

$\Rightarrow \quad \sin \alpha = \frac{n_1}{n_2} \cos\left\{\sin^{-1}\left(\frac{n_2}{n_1}\right)\right\}$

or $\quad \alpha = \sin^{-1}\left[\frac{n_1}{n_2} \cos\left\{\sin^{-1}\left(\frac{n_2}{n_1}\right)\right\}\right]$

10. **(b)**



The incident and emergent ray OA || BC & OD || EF of a glass slab are parallel therefore, the divergence angle remains the same.

11. **(a, b)** We have

$$\frac{\sin i}{\sin r'} = \frac{\mu_r}{\mu_d} \Rightarrow \frac{\mu_r}{\mu_d} - \frac{\sin r}{\sin r'} = \frac{\sin r}{\cos r} = \tan r = \tan i$$

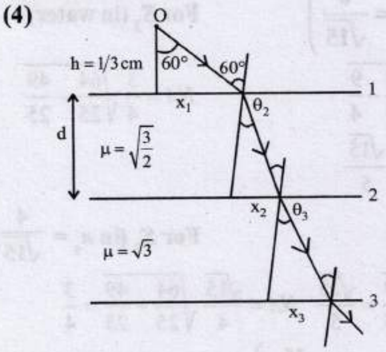
$[\because r + r' = 90°]$

So, $\sin C = {}^{d}\mu_r = \tan i$

$$C = \sin^{-1}(\tan i) = \sin^{-1}(\tan r)$$

12. **(a)** **Frequency (v)** does not change with medium
Glass slab is an optically denser medium, the velocity of light decreases and therefore the wavelength in glass decreases.

13. **(4)**



$$x_1 = \frac{1}{3} \times \tan 60° = \frac{1}{\sqrt{3}} \text{cm}$$

and, **at interface-1**

$$1 \sin 60° = \sqrt{\frac{3}{2}} \sin\theta_2 \Rightarrow \theta_2 = 45°$$

$$\text{So, } x_2 = d \tan 45° = \frac{\sqrt{3}-1}{2}$$

at interface 2:

$$\text{Again, } \sqrt{\frac{3}{2}} \sin 45° = \sqrt{3} \sin\theta_3$$

$$\Rightarrow \theta_3 = 30°$$

$$\text{So, } x_3 = d \tan 30° = \frac{\sqrt{3}-1}{2\sqrt{3}}$$

$$\text{So, } x_1 + x_2 + x_3 = \frac{1}{\sqrt{3}} + \frac{\sqrt{3}-1}{2}\left(1 + \frac{1}{\sqrt{3}}\right)$$

$$= \frac{1}{\sqrt{3}} + \frac{(\sqrt{3}-1)(\sqrt{3}+1)}{2\sqrt{3}} = \frac{1}{\sqrt{3}} + \frac{1}{\sqrt{3}} = \frac{2}{\sqrt{3}} \text{cm}$$

So 1 unit shift light ray by $\frac{2}{\sqrt{3}}$, therefore to shift light ray

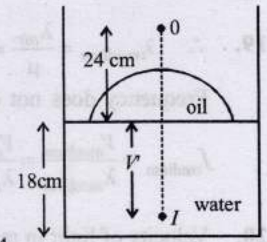
by $\ell = \frac{8}{\sqrt{3}}$, no. of units needed, $n = \dfrac{\frac{8}{\sqrt{3}}}{\frac{2}{\sqrt{3}}} = 4$

14. **(8)** Here, $n \sin\theta = (n - m\Delta n) \sin 90°$
$\Rightarrow n \times \sin 30° = [n - m \times 0.1] \sin 90°$
$\therefore 1.6 \times \sin 30° = 1.6 - m \times 0.1 \quad \therefore m = 8$

15. **(2)** For the convex spherical refracting surface i.e., air-oil interface. Since oil is acting as thin lens we will ignore its dimension

$u = -24$ cm, $v = ?$, $\mu_1 = 1$, $\mu_2 = \frac{7}{4}$ and $R = 6$ cm

$$\frac{-\mu_1}{u} + \frac{\mu_2}{v} = \frac{\mu_2 - \mu_1}{R}$$



$$\therefore \frac{-1}{(-24)} + \frac{7/4}{v} = \frac{\frac{7}{4} - 1}{6}$$

$\therefore v = 21$ cm

This image will act as object for the water-oil interface

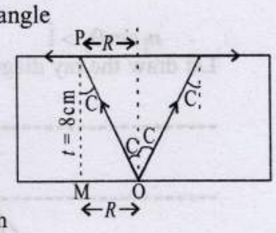
$u = 21$ cm, $v = v'$, $\mu_1 = \frac{7}{4}$, $\mu_2 = \frac{4}{3}$ and $R = \infty$

$$\frac{\frac{-7}{4}}{+21} + \frac{\frac{4}{3}}{V'} = 0$$

$\therefore V' = 16$ cm.

Therefore the distance of the image from the bottom of the tank = 18 – 16 = 2 cm.

16. **(6)** In the figure, C = critical angle



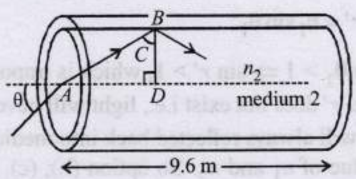
In ΔPOM, $\tan C = \frac{OM}{PM} = \frac{R}{8}$

$$\because \sin C = \frac{1}{\mu} = \frac{3}{5} \quad \therefore \tan C = \frac{3}{4}$$

$$\therefore \frac{R}{8} = \frac{3}{4} \quad \therefore R = \frac{3}{4} \times 8 = 6 \text{cm}$$

17. **(50)** Let 'C' be the critical angle.
1.5 sin C = 1.44 sin 90°

$$\Rightarrow \sin C = \frac{1.44}{1.50}$$



$$\text{But } \sin C = \frac{AD}{AB}$$

$$\therefore \frac{AD}{AB} = \frac{1.44}{1.50} \quad \therefore AB = \frac{1.50}{1.44} AD$$

If we replace AD by 9.6 then total length travelled by light

$$= \frac{1.50}{1.44} \times 9.6 = 10\text{m}$$

Maximum, time taken by a ray to exit the plane

$$t = \frac{d}{v_2} = \frac{d}{C/n_2}$$

$$\therefore t = \frac{10}{3 \times 10^8 / 1.5} = 5 \times 10^{-8} \text{s} = 50 \times 10^{-9} \text{s}$$

Comparing it with t × 10^{-9} s we get, t = 50.00

18. Frequency does not change with medium

$$f = \frac{c}{\lambda} = \frac{3 \times 10^8}{6000 \times 10^{-10}} = 5 \times 10^{14} \text{Hz}$$

$$\text{Wavelength in medium, } \lambda^1 = \frac{\lambda}{\mu} = \frac{6000Å}{1.5} = 4000Å$$

19. $\therefore \quad \lambda_{medium} = \frac{\lambda_{air}}{\mu} = \frac{6000}{1.5} = 4000\text{Å}$

Frequency does not change with the medium

$$f_{medium} = \frac{V_{medium}}{\lambda_{medium}} = \frac{V_{air}}{\lambda_{air}} = \frac{3\times10^8}{6\times10^{-7}} = 5\times10^{14}\text{Hz}$$

20. Velocity of light in medium, $V = \frac{c}{\mu}$

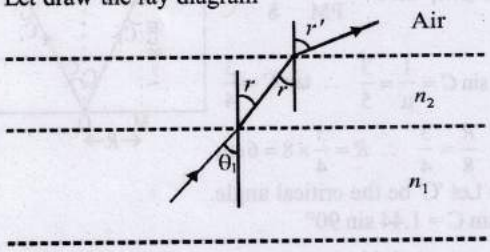
$$= \frac{3\times10^8}{1.5} = 2\times10^8 \text{ m/s};$$

Wavelength, $\lambda = \frac{v}{f} = \frac{2\times10^8}{5\times10^{14}} = 4\times10^{-7} m$

21. **(b, c, d)** Given, $\sin\theta_1 > \frac{1}{n_1}$

$n_1 \sin\theta_1 > 1$

Let draw the ray diagram



So, by Snell's law,

$n_1 \sin\theta_1 = n_2 \sin r = 1 \times \sin r'$

$\Rightarrow \quad \sin r' = n_1 \sin\theta_1$

As $n_1 \sin\theta_1 > 1 \Rightarrow \sin r' > 1$, which is impossible. Therefore, r' does not exist i.e., light will never enter the air. So, light will always reflected back into medium 1 whatever is the value of n_1 and n_2. So option (b), (c), (d) is correct.

22. **(a, c, d)** From Snell's law, $n_1 \sin\theta_i = n_2 \sin\theta_f$ [$\because$ 1 and 2 interfaces are parallel]

l depends on the refractive index of transparent slab $n(z)$ but not on n_2. But θ_f depends, on n_2.

Because lateral displacement (l) is possible due μ of slab and angle of incidence θ_i.

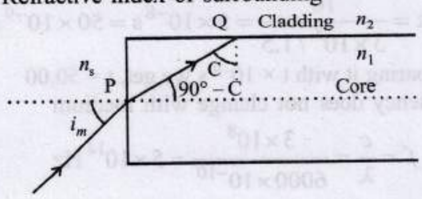
23. **(c, d)** For the ray undergoes total internal reflection

Angle of incidence, $i >$ critical angle, θ_c

or $\sin 45° > \frac{1}{n}$ or $\frac{1}{\sqrt{2}} > \frac{1}{n}$ or $n > \sqrt{2}$ or $n > 1.414$.

24. **(a, c)** Using Snell's law at P; $n_s \sin i_m = n_1 \sin(90° - C)$ ...(i)

n_s = Refractive index of surrounding



Also $\sin C = \frac{n_2}{n_1}$ $\quad \therefore \quad \cos C = \sqrt{1-\left(\frac{n_2}{n_1}\right)^2}$

Now from eq. (i)

Numerical aperture,

$$NA = \sin i_m = \frac{n_1}{n_s}\cos C = \frac{n_1}{n_s}\sqrt{1-\frac{n_2^2}{n_1^2}}$$

$$\therefore \quad NA = \frac{\sqrt{n_1^2 - n_2^2}}{n_s}$$

For S_1 (in air)

$$NA = \sqrt{\frac{45}{16} - \frac{9}{4}} = \frac{3}{4}$$

For S_1 (in water)

$$NA = \frac{3}{4}\sqrt{\frac{45}{16} - \frac{9}{4}} = \frac{9}{16}$$

For s_1 $\left(\text{in } n_s = \frac{6}{\sqrt{15}}\right)$

$$NA = \frac{\sqrt{15}}{6}\sqrt{\frac{45}{16} - \frac{9}{4}} = \frac{3\sqrt{15}}{24} = \frac{3}{4}\frac{\sqrt{15}}{5}$$

For S_2 (in water)

$$NA = \frac{3}{4}\sqrt{\frac{64}{25} - \frac{49}{25}}$$

For S_2 (in air)

$$NA = \sqrt{\frac{64}{25} - \frac{49}{25}} = \frac{\sqrt{15}}{5}$$

For S_2 (in $n_s = \frac{4}{\sqrt{15}}$)

$$NA = \frac{\sqrt{15}}{4}\sqrt{\frac{64}{25} - \frac{49}{25}} = \frac{3}{4}$$

For S_2 $\left(\text{in } n_s = \frac{16}{3\sqrt{15}}\right)$

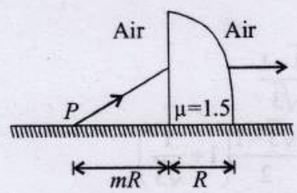
$$NA = \frac{3\sqrt{15}}{16}\sqrt{\frac{64}{25} - \frac{49}{25}} = \frac{9}{16}$$

25. **(d)** Numerical aperture, $NA = \frac{1}{n_s}\sqrt{n_1^2 - n_2^2}$

Here, $NA_2 < NA_1 \Rightarrow i_{m_2} < i_{m_1}$

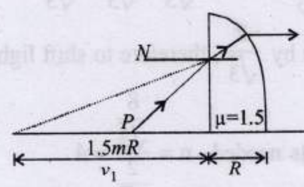
$\therefore$ Numerical aperture, of combined structure is equal to the smaller value of the two numerical apertures.

26. Apply formula for refraction at a spherical surface when light travels from air ($\mu_1 = 1$) to medium (μ_2).



$$\frac{\mu_2}{v} - \frac{\mu_1}{u} = \frac{\mu_2 - \mu_1}{r}$$

First refraction occurs on plane surface for which $r = \infty$.



$$\therefore \quad \frac{1.5}{v_1} - \frac{1}{-mR} = \frac{1.5-1}{\infty}$$

or $\quad v_1 = -(1.5mR)$

Then, on curved surface

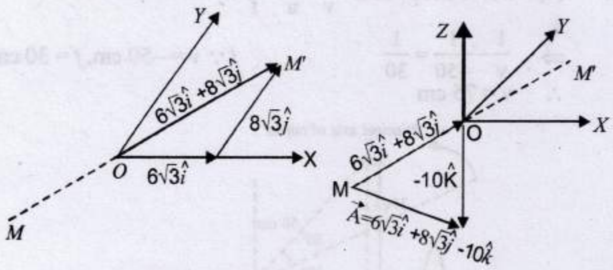
$$\frac{\mu_1}{v_2} - \frac{\mu_2}{u_2} = \frac{\mu_1 - \mu_2}{r}, \text{ where } u_2 = v_1 + R, v_2 = \infty$$

or $\quad \frac{1}{\infty} - \frac{1.5}{-(1.5mR + R)} = \frac{1-1.5}{-R}$

or $\dfrac{1.5}{R(1.5m+1)}=\dfrac{0.5}{R}$ or $3=1.5m+1$

or $1.5m=2$ or $m=\dfrac{4}{3}$

27. The x - y plane is the boundary between medium-1 and medium-2



(Fig. a) (Fig. b)

Figure (a) shows vector $OM'=6\sqrt{3}\text{i}+8\sqrt{3}\text{j}$

Figure (b) shows vector $\vec{A}=6\sqrt{3}\hat{i}+8\sqrt{3}\hat{j}-10\hat{k}$

The perpendicular to line MOM' is Z-axis which has a unit vector of $\hat{k}$.

Angle between vector $\overline{MP}$ and $\overline{OP}$ can be found by dot product.

$\overline{MP}\,.\,\overline{OP}=(MP)\,(OP)\cos i$

$$\frac{(6\sqrt{3}\hat{i}+8\sqrt{3}\hat{j}-10\hat{k}).(-\hat{k})}{\sqrt{(6\sqrt{3})^2+(8\sqrt{3})^2+(-10)^2}+\sqrt{(-1)^2}}=\cos i$$

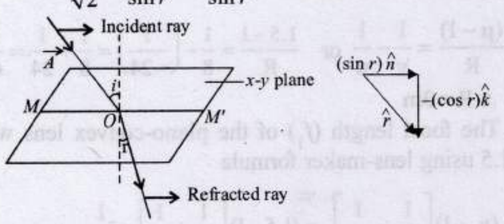
$\Rightarrow\ i=60°$

Unit vector in the direction of MOM' from fig. (a)

$$\hat{n}=\frac{6\sqrt{3}\hat{i}+8\sqrt{3}\hat{j}}{[(6\sqrt{3})^2+(8\sqrt{3})^2]^{1/2}},\ \hat{n}=\frac{3}{5}\hat{i}+\frac{4}{5}\hat{j}$$

To find the angle of refraction, we use Snell's law

$$\frac{\sqrt{3}}{\sqrt{2}}=\frac{\sin i}{\sin r}=\frac{\sin 60°}{\sin r}\Rightarrow r=45°$$



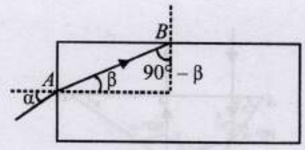
Now, $\hat{r}=(\sin r)\hat{n}-(\cos r)\hat{k}$

$$=(\sin 45°)\left[\frac{3}{5}\hat{i}+\frac{4}{5}\hat{j}\right]-(\cos 45°)\hat{k}$$

$$\therefore\ \hat{r}=\frac{1}{5\sqrt{2}}[3\hat{i}+4\hat{j}-5\hat{k}]$$

28. The angle of incidence at the curved surface of the cylindrical rod is given by $(90°-\beta)$.

The light entering the rod does not emerge from the curved surface of the rod, if the angle $(90°-\beta)$ is greater than the critical angle C.



i.e., $\mu\le\dfrac{1}{\sin C}$ where C is the critical angle.

Here, $C=90°-\beta\Rightarrow\mu\le\dfrac{1}{\sin(90°-\beta)}\Rightarrow\mu\le\dfrac{1}{\cos\beta}$

As a limiting case, $\mu=\dfrac{1}{\cos\beta}$ ... (i)

Applying Snell's law at A

$$\mu=\frac{\sin\alpha}{\sin\beta}\Rightarrow\sin\beta=\frac{\sin\alpha}{\mu}\quad\text{... (ii)}$$

If $\alpha=\pi/2$ as a limiting case on planar end of rod

$$\therefore\ \sin\beta=\frac{\sin\alpha}{\mu}=\frac{\sin\pi/2}{\mu}\Rightarrow\mu=\frac{1}{\sin\beta}\quad\text{... (iii)}$$

From eq. (i) and (iii), $\sin\beta=\cos\beta$

or $\beta=45°$

$$\therefore\ \mu=\frac{1}{\cos 45°}=\frac{1}{1/\sqrt{2}}\ \text{or}\ \mu=\sqrt{2}$$

$\therefore$ The least value of the refractive index of rod for light entering the rod and not leaving it from the curved surface is $\sqrt{2}$.

29. (i) According to question, a parallel beam of light is travelling in water by a spherical bubble of radius 2 nm. Initially the object is in denser medium and $u=\infty$ using the formula of refraction at a spherical surface for AB see diagram below.

$$-\frac{\mu_2}{u}+\frac{\mu_1}{v}=\frac{\mu_1-\mu_2}{R}\Rightarrow\frac{-4/3}{-\infty}+\frac{1}{v}=\frac{1-4/3}{2}$$

$\therefore\ v=-6$ mm

This is the position of the image due to refraction at the first surface i.e., at 6 mm left of first surface. This image will behave as a virtual object for the refraction at the second surface.

$u=-6-4=-10$ mm

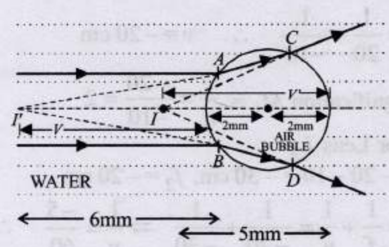
Again using the formula of refraction at a spherical surface for CD

$$-\frac{\mu_1}{u'}+\frac{\mu_2}{v'}=\frac{+\mu_2-\mu_1}{R},\ -\frac{1}{10}+\frac{4/3}{v'}=\frac{\frac{4}{3}-1}{-2}$$

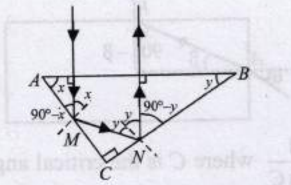
$\therefore\ v'=-5$ mm.

i.e., Final image, is formed 5 mm to the left of second surface.

(ii) Ray diagram showing the positions of both the images.



30. (i) Let x is the incident angle for reflection at face AC of the prism. For total internal reflection $x>i_C$ (critical angle)



Let y be the incident angle of the ray on face CB of the prism. For total internal reflection

$y > i_C$

$\therefore \quad x + y > 2i_C$

But $x = \angle A$ and $y = \angle B$ (from geometry)

$\therefore \quad x + y = 90°$

$\Rightarrow \quad 90 > 2i_C \Rightarrow i_C < 45°$

$\therefore$ Minimum value of refractive index n of the medium for this to be possible.

$$n = \frac{1}{\sin i_C} = \frac{1}{\sin 45°} = \sqrt{2}$$

(ii) For $n = \frac{5}{3} \Rightarrow \sin i_C' = \frac{1}{\mu} = \frac{1}{5/3} = \frac{3}{5} \Rightarrow i_C' = 37°$

$y = 30°$ (Given) $\therefore x = 60°$

$x > i_C'$ but $y < i_C'$

$\therefore$ Total internal reflection will take place on face AC but not on CB.

Topic-3: Refraction at Curved Surface, Lenses and Power of Lens

1. **(b)** From lens maker's formula, focal length of convex lens (f_1)

$$\frac{1}{f_1} = (\mu - 1)\left[\frac{1}{R_1} - \frac{1}{R_2}\right] = (1.5-1)\left[\frac{1}{20} - \left(\frac{1}{-20}\right)\right] \frac{1}{f_1} = \frac{1}{20}$$

$\therefore f_1 = +20$ cm

Similarly, focal length of concave lens (f_2)

$$\frac{1}{f_2} = (\mu - 1)\left[\frac{1}{R_1} - \frac{1}{R_2}\right]$$

$$\Rightarrow \frac{1}{f_2} = (1.5-1)\left[-\frac{1}{20} - \frac{1}{20}\right] = \frac{1}{-20}$$

$\therefore f_2 = -20$ cm

Now for lens L_1

$$\frac{1}{v} - \frac{1}{u} = \frac{1}{f} \Rightarrow \frac{1}{v} = \frac{1}{f} + \frac{1}{u}$$

$$\Rightarrow \frac{1}{v} = \frac{1}{20} + \frac{1}{-10} \quad \therefore \quad v = -20 \text{ cm}$$

$\therefore$ Magnification $M_1 = \frac{v}{u} = \frac{-20}{-10} = 2$

Again for Lens L_2

$u = -20 - 10 = -30$ cm, $f_2 = -20$ cm

$$\frac{1}{v} = \frac{1}{f} + \frac{1}{u} = \frac{1}{-20} + \frac{1}{-30} \Rightarrow \frac{1}{v} = \frac{-5}{60} \quad \therefore \quad v = -12 \text{ cm}$$

So, magnification,

$$m_2 = \frac{v}{u} = \frac{-12}{-30} = \frac{2}{5}$$

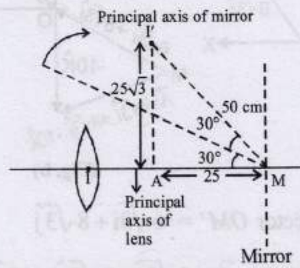
$\therefore$ Net magnification,

$$m = m_1 m_2 = 2 \times \frac{2}{5} = \frac{4}{5} = 0.8$$

2. **(c)** *For convex lens* $\frac{1}{v} - \frac{1}{u} = \frac{1}{f}$

$\Rightarrow \frac{1}{v} - \frac{1}{-50} = \frac{1}{30}$ $\quad (\because v = -50 \text{ cm}, f = 30 \text{ cm})$

$\therefore \quad v = 75$ cm



The image formed by convex lens acts as an object for mirror.

For minor $\frac{1}{v} + \frac{1}{u} = \frac{1}{f} \therefore \frac{1}{v} + \frac{1}{25} = \frac{1}{50} \therefore v = -50$ cm

The image I would have formed as shown had the mirror been straight. But here the mirror is tilted by 30°. Therefore the image will be tilted by 60° and will be formed at A.

Here MA = 50 cos 60° = 25 cm (x-coordinate of the image)

and I'A = 50 sin 60° = $25\sqrt{3}$ cm (y-coordinate of the image)

Hence, coordinate of the point at which image is formed $(25, 25\sqrt{3})$.

3. **(c)** Here $\mu = \frac{\lambda_{air}}{\lambda_{medium}} = \frac{1}{\frac{2}{3}} = \frac{3}{2} = 1.5$

Also $m = \frac{v}{u} = -\frac{1}{3}$ $(\because v = 8m)$

$\therefore u = -24$ cm.

For a plano-convex lens

$$\frac{1}{f} = \frac{(\mu-1)}{R} = \frac{1}{v} - \frac{1}{u} \text{ or } \frac{1.5-1}{R} = \frac{1}{8} - \left(\frac{1}{-24}\right) = \frac{1}{8} + \frac{1}{24} = \frac{1}{6}$$

$\therefore R = 3m$

4. **(b)** The focal length (f_1) of the plano-convex lens with $n = 1.5$ using lens-maker formula

$$\frac{1}{f_1} = (n_1 - 1)\left[\frac{1}{R_1} - \frac{1}{R_2}\right] = (1.5-1)\left[\frac{1}{14} - \frac{1}{\infty}\right] = \frac{1}{28}$$

The focal length (f_2) of the plano-convex lens with $n = 1.2$

$$\frac{1}{f_2} = (n_2 - 1)\left[\frac{1}{R_1} - \frac{1}{R_2}\right] = (1.2-1)\left[\frac{1}{\infty} - \frac{1}{-14}\right] = \frac{1}{70}$$

Focal length F of the combination

$$\frac{1}{F} = \frac{1}{f_1} + \frac{1}{f_2} = \frac{1}{20}$$

Now, applying lens formula for the combination of lens

$\frac{1}{V}-\frac{1}{U}=\frac{1}{F} \Rightarrow \frac{1}{V}-\frac{1}{-40}=\frac{1}{20}$ [Given μ = 40 cm]

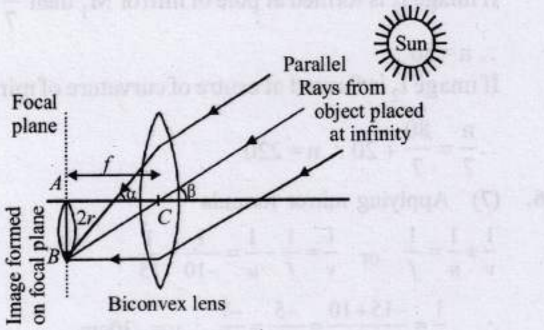
$\therefore \quad V = 40$ cm

5. **(b)** From the figure in ΔABC, $\tan\beta = \frac{AB}{AC}$

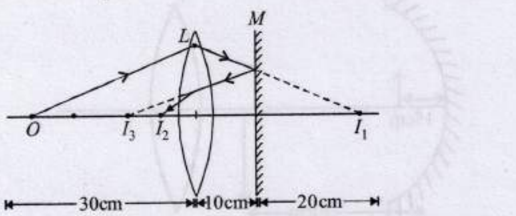
$\Rightarrow AB = AC\tan\beta$

$\Rightarrow 2r = f\tan\beta \;\therefore\; r = \frac{f}{2}\tan\beta$

$\therefore$ Area of image formed by sun $= \pi r^2 = \pi\frac{\tan^2\beta}{2}f^2 \propto f^2$



6. **(b)** Focal length of the biconvex lens L is 15 cm. A small object is placed at a distance of 30 cm from the lens i.e. at a distance of 2f. Therefore the image should form at 30 cm from the lens at I_1.



The image I_1 acts as a virtual object for the mirror. The mirror forms an image I_2 at a distance of 20 cm in front of it. The image I_2 acts as an object for the lens.

Here, $u = +10$ cm, $f = +15$cm

Applying lens formula $\frac{1}{v}-\frac{1}{u}=\frac{1}{f}$

$\Rightarrow \frac{1}{v}-\frac{1}{10}=\frac{1}{15}$

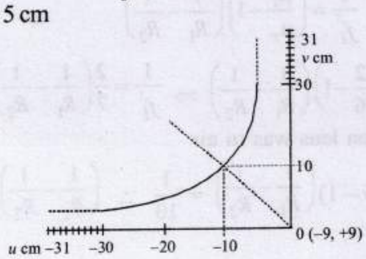
$\Rightarrow \frac{1}{v}=\frac{1}{15}+\frac{1}{10}=\frac{25}{150} \quad \therefore \quad v = 6$ cm.

Therefore final real image is formed at a distance of 16 cm from the plane mirror.

7. **(c)** From the graph $u = -10$ cm; $v = 10$ cm $\Delta u = \Delta v = 0.1$

From lens formula, $\frac{1}{f}=\frac{1}{v}-\frac{1}{u}=\frac{1}{10}-\frac{1}{-10}$

$\therefore \quad f = 5$ cm



Differentiating lens formula,

$\frac{1}{f}=\frac{1}{v}-\frac{1}{u} \Rightarrow \frac{\Delta f}{f^2}=\frac{\Delta v}{v^2}+\frac{\Delta u}{u^2}$ (for maximum error in f)

$\Rightarrow \frac{\Delta f}{25}=\frac{0.1}{(10)^2}+\frac{0.1}{(10)^2} \Rightarrow \Delta f = 25\times 0.1\times 2\times 0.01 = 0.05$

$\therefore$ Focal length, $f \pm \Delta f = (5.00 \pm 0.05)$ cm.

8. **(b)** The focal length f of the equivalent mirror when lens is silvered,

$\frac{1}{f}=\frac{2}{f_1}+\frac{1}{f_m}=\frac{2}{15}+\frac{1}{\infty} \Rightarrow f=\frac{15}{2}$cm

Since f has a positive value, the combination behaves as a converging mirror.

Here $u = -20$cm, $f = -\frac{15}{2}$cm, $v = ?$

According to mirror formula $\frac{1}{v}+\frac{1}{u}=\frac{1}{f}$

$\Rightarrow \frac{1}{v}-\frac{1}{-20}=\frac{1}{-15/2} \Rightarrow v = -12$ cm

Negative sign indicates that the image is 12 cm in front or left of mirror.

9. **(a)** Focal length of concave lens, $f_2 = \frac{3}{2}f_1$

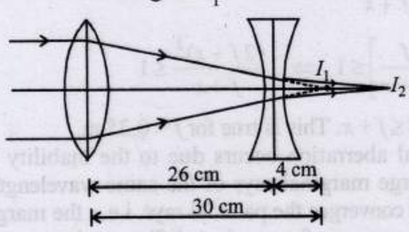
f_1 = focal length of convex lens.

$\frac{1}{f}=\frac{1}{f_1}-\frac{1}{f_2} \Rightarrow \frac{1}{30}=\frac{1}{f_1}-\frac{2}{3f_1}$

$\therefore \quad f_1 = 10$ cm and $f_2 = -\frac{3}{2}\times 10 = -15$ cm

10. **(b)** The image formed by convex lens, I_1 acts as virtual object for concave lens. Concave lens forms the image of I_1 at I_2.

Size of the image at $I_1 = 2$ cm



For concave lens, $\frac{1}{v}-\frac{1}{u}=\frac{1}{f}$

or, $\frac{1}{v}-\frac{1}{4}=-\frac{1}{20}$ or $\frac{1}{v}=-\frac{1}{20}+\frac{1}{4}=\frac{4}{20}=\frac{1}{5}$

or $v = 5$ cm = Distance of I_2 from concave lens.

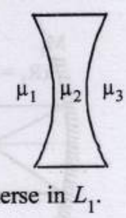
$\therefore$ Magnification, $m=\frac{v}{u}=\frac{\text{size of image}}{\text{size of object}}=\frac{5}{4}$

or $\frac{\text{size of image}}{2}=1.25$

$\therefore$ Size of image due to concave lens = 2.5 cm

11. **(d)** The Lens maker formula,

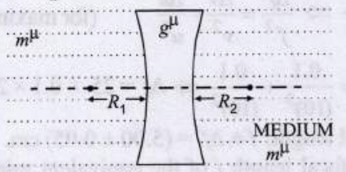
$\frac{1}{f}=\left(\frac{\mu_2}{\mu_1}-1\right)\left(\frac{1}{R_1}-\frac{1}{R_2}\right)$



If $\mu_2 > \mu_1$, the concave lens maintains its nature otherwise the nature of the lens will be reversed.

So, the lens should be filled with L_2 and immerse in L_1.

12. **(a)** For concave lens as shown in figure, in this case $R_1 = -R$ and $R_2 = +R$



From lens maker formula $\frac{1}{f} = \left({}^{m}_{g}\mu - 1\right)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$

$${}^{m}_{g}\mu = \frac{{}_g\mu}{{}_m\mu} = \frac{1.5}{1.75}$$

$$\therefore \quad \frac{1}{f} = \left(\frac{1.5}{1.75} - 1\right)\left(-\frac{1}{R} - \frac{1}{R}\right) = +\frac{0.25 \times 2}{1.75R}$$

or, $f = +3.5R$

Hence lens behaves as convergent lens of focal length $f = 3.5$R

13. **(c)** A convex mirror and a concave lens always produce virtual image. Which cannot be taken on the screen. Therefore, option (b) and (d) are not correct. The image formed by a convex lens is diminished when the object is placed beyond $2f$.

Let $u = 2f + x$

$$\text{Using} \quad \frac{1}{v} - \frac{1}{u} = \frac{1}{f} \Rightarrow \frac{1}{v} - \frac{1}{-(2f+x)} = \frac{1}{f}$$

$$\Rightarrow \frac{1}{v} = \frac{1}{f} + \frac{1}{2f+x} \frac{2f+x-f}{f(2f+x)} = \frac{(f+x)}{f(2f+x)}$$

But $u + v = 1$ (given)

$$(2f+x) + \frac{f(2f+x)}{f+x} \le 1$$

$$2f + x\left[1 + \frac{f}{f+x}\right] \le 1 \Rightarrow \frac{(2f+x)^2}{f+x} \le 1$$

$\therefore \quad (2f+x)^2 \le f + x$. This is true for $f < 0.25$ m.

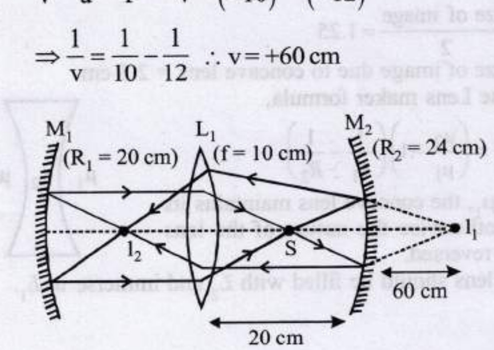
14. **(c)** Spherical aberration occurs due to the inability of a lens to converge marginal rays of the same wavelength to the focus as it converges the paraxial rays. i.e., the marginal and paraxial says are focussed at different places on the axis of the lens and therefore the image so formed is blurred. This aberration can be reduced by using a circular annular mask over the lens.

15. **(80 or 150 or 220)**

From reflection from mirror M_2

$$\frac{1}{v} + \frac{1}{u} = \frac{1}{f} \Rightarrow \frac{1}{v} + \frac{1}{(-10)} = \frac{1}{(-12)}$$

$$\Rightarrow \frac{1}{v} = \frac{1}{10} - \frac{1}{12} \quad \therefore v = +60 \text{ cm}$$



For refraction from lens, L

$$\frac{1}{v} - \frac{1}{u} = \frac{1}{f} \text{ or, } \frac{1}{v} - \frac{1}{(-80)} = \frac{1}{10}$$

$\therefore v = +\frac{80}{7}$ (For image I_2)

This image should be at focus of M_1

$$\therefore \frac{20}{2} + \frac{80}{7} = \frac{n}{7} \Rightarrow \frac{300}{14} = \frac{n}{7} \therefore n = 150$$

If image I_2 is formed at pole of mirror M_1 then $\frac{n}{7} = \frac{80}{7}$

$\therefore n = 80$

If image I_2 is formed at centre of curvature of mirror M_1

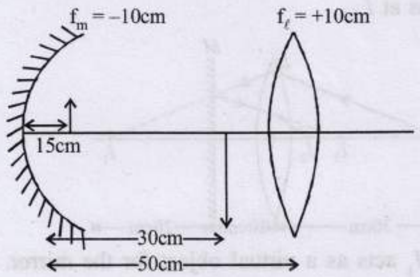
$$\therefore \frac{n}{7} = \frac{80}{7} + 20 \therefore n = 220$$

16. **(7)** Applying mirror formula

$$\frac{1}{v} + \frac{1}{u} = \frac{1}{f} \quad \text{or} \quad \frac{1}{v} = \frac{1}{f} - \frac{1}{u} = \frac{1}{-10} + \frac{1}{15}$$

$$\therefore \quad \frac{1}{v} = \frac{-15+10}{150} = \frac{-5}{150} = \frac{-1}{30} \quad \therefore v = -30\text{cm}$$

And magnification, $m_1 = -\frac{v}{u} = -\frac{-30}{-15} = -2$



Now for refraction from lens, $u = -(50 - 30) = -20$ cm

$$\frac{1}{v} - \frac{1}{u} = \frac{1}{f_l} \Rightarrow \frac{1}{v} - \frac{1}{-20} = \frac{1}{10}$$

$$\frac{1}{v} = \frac{1}{10} - \frac{1}{20} = \frac{1}{20} \quad \therefore \ v = 20 \text{ cm}$$

Magnification, $m_2 = \frac{v}{u} = \frac{20}{-20} = -1$

Magnification produced by the combination,

$$M_1 = m_1 \times m_2 = (-2) \times (-1) = 2$$

Again, when system is kept in a medium of refractive index 7/6.

There is no change for mirror in this case,

$$\text{For lens, } \frac{1}{f_l'} = \left(\frac{\mu_l}{\mu_s} - 1\right)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$$

$$\frac{1}{f_l'} = \left(\frac{3/2}{7/6} - 1\right)\left(\frac{1}{R_1} - \frac{1}{R_2}\right) \Rightarrow \frac{1}{f_l'} = \frac{2}{7}\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$$

Also, when lens was in air

$$\frac{1}{f_l} = (1.5 - 1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right) = \frac{1}{10} \quad \therefore \quad \left(\frac{1}{R_1} - \frac{1}{R_2}\right) = \frac{1}{5}$$

Using this result in eqn. (i), we get

$\frac{1}{f_l'}=\frac{2}{7}\times\frac{1}{5}\quad\therefore\ f_l'=\frac{35}{2}$ cm

Again using lens formula, $\frac{1}{v}-\frac{1}{u}=\frac{1}{f_l'}$

$\frac{1}{v}-\frac{1}{-20}=\frac{2}{35}\Rightarrow\frac{1}{v}=\frac{2}{35}-\frac{1}{20}=\frac{1}{140}\quad\therefore v=140$ cm

Magnification, $m_2'=\frac{v}{u}=\frac{140}{-20}=-7$

Magnification produced by the combination,

$M_2=m_1\times m_2'=(-2)\times(-7)=14$

$\therefore\quad\left|\frac{M_2}{M_1}\right|=\frac{14}{2}=7$

17. **(6)** When $u=-25$ cm

$$\frac{1}{v}=\frac{1}{f}+\frac{1}{u}=\frac{1}{20}+\frac{1}{-25}=\frac{1}{100}\Rightarrow v=100\text{ cm}$$

$$m_{25}=\frac{-v}{u}=\frac{-100}{-25}=4$$

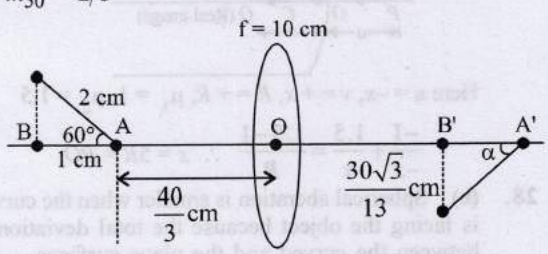
When $u=-50$ cm

$$\frac{1}{v}=\frac{1}{f}+\frac{1}{u}=\frac{1}{20}+\frac{1}{-50}=\frac{3}{100}\Rightarrow v=\frac{100}{3}\text{ cm}$$

$$m_{50}=\frac{-v}{u}=\frac{-1000}{3}\times\frac{-1}{50}=\frac{2}{3}$$

So, $\frac{m_{25}}{m_{50}}=\frac{4}{2/3}=6$

18. **(6)**



By lens formula, $\frac{1}{v}=\frac{1}{f}+\frac{1}{u}\Rightarrow v=\frac{uf}{f+u}$

So, $OA'=\frac{-\frac{40}{3}\times10}{-\frac{40}{3}+10}=40$ cm

and, $OB'=\frac{-\frac{43}{3}\times10}{-\frac{43}{3}+10}=\frac{430}{13}$ cm

So, $A'B'=OA'-OB'=40-\frac{430}{13}=\frac{90}{13}$ cm

Therefore, $\tan\alpha=\frac{\frac{30\sqrt{3}}{13}}{\frac{90}{13}}=\frac{1}{\sqrt{3}}\Rightarrow\alpha=30°=\frac{\pi}{6}$

So, n = 6

19. When two lenses are in contact, $P=P_1+P_2=10$

When two lenses are separated by a distance d(d = 0.25m)

$P=P_1+P_2-(0.25)\,P_1P_2=6$

From these two expressions, we get

$P_1P_2=16\text{m}^{-2}$

$P_1-P_2=\sqrt{(P_1+P_2)^2-4P_1P_2}$

$=\sqrt{(10^{-1})^2-4(16^{-1})}=6\text{m}^{-1}$

$\therefore\quad P_1=8\text{m}^{-1}$ and $P_2=2\text{ m}^{-1}$

$\therefore\ f_1=\frac{1}{P_1}=\frac{1}{8}\text{m}=0.125\text{m}$ and $f_2=\frac{2}{P_2}=\frac{1}{2}\text{m}=0.5\text{m}$

20. ${}^{m}_{g}\mu=\frac{{}^{g}\mu}{{}^{m}\mu}=\frac{1.5}{4/3}=1.125$

using $\frac{1}{f}=(\mu-1)\left(\frac{1}{R_1}-\frac{1}{R_2}\right)$ [Lens-maker's formula]

$\frac{1}{15}=(1.5-1)\left(\frac{1}{R_1}-\frac{1}{R_2}\right)$ ....(i)

and $\frac{1}{f'}=(1.125-1)\left(\frac{1}{R_1}-\frac{1}{R_2}\right)$ ....(ii)

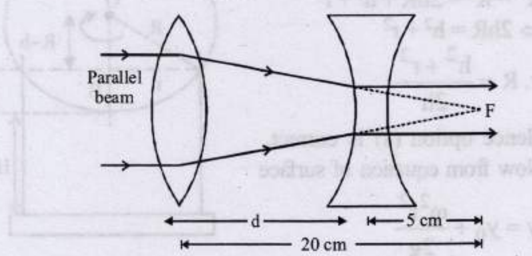
Dividing eq. (i) by (ii)

$\frac{f'}{15}=\frac{1.5-1}{1.125-1}=\frac{0.5}{0.125}=4$

$\therefore\quad f'=60$ cm

21. From the diagram it is clear that the focus of both the lenses should coincide at F

$\therefore d=20-5=15$ cm.



22. **True**

$\frac{1}{F}=\frac{1}{f_1}+\frac{1}{f_2}\ \Rightarrow\ \frac{1}{F}=\frac{1}{-15}+\frac{1}{30}=\frac{-2+1}{30}$

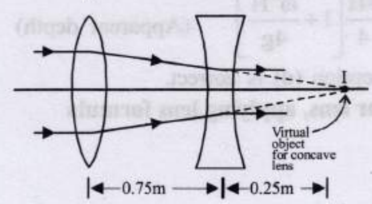
$\therefore\quad F=-30$ cm.

Hence combination of lenses behaves as a concave lens of focal length 30 cm.

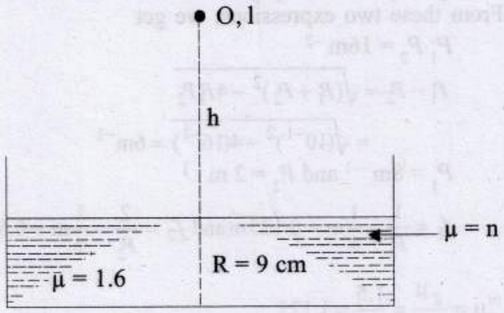
Since $F_v<F_r$, violet colour is deviated more than red colour when white light passes through a lens or prism

$\therefore$ One sees coloured pattern with violet colour at the outer edge.

23. **False** The image formed by the convex lens at the focus of the concave lens. Therefore I will act as a virtual object for concave lens and image will be formed at infinity. Hence after refraction from both the lenses, rays will become parallel to the optic axis.



24. **(a,b)** The image of a point object o at a height of h in the figure is formed onto itself i.e., object and image are at same distance from equivalent lens so $h = 2f_{eq}$



$$\Rightarrow \frac{1}{f_{net}} = 2\left(\frac{1}{f_{liq}}\right) + 2\left(\frac{1}{f_{lens}}\right) + \left(\frac{-1}{f_{mirror}}\right) \quad ....(i)$$

$$-\frac{1}{f_{net}} = 2\left(\frac{n-1}{-9}\right) + 2\left(\frac{0.6}{9}\right) + \left(\frac{-1}{\infty}\right) \quad ...(ii)$$

From (i) and (ii)

$$\frac{1}{h/2} = \frac{1.2}{9} + \frac{2(1-n)}{9} \Rightarrow \frac{2}{h} = \frac{3.2-2n}{9} \text{ or, } h = \frac{9}{1.6-n} \text{ cn}$$

Therefore for (a) n = 1.42 h = 50cm (b) n = 1.35, h = 36cm (c) n = 1.45, h = 60 cm (d) n = 1.48, h = 75cm

so option (a) and (b) are correct.

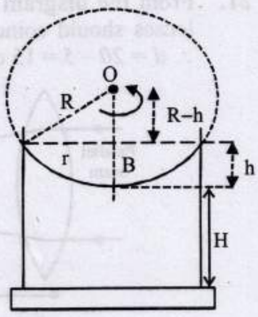
25. **(a, d)** From figure, in ΔOAB

$R^2 = (R - h)^2 + r^2$

$R^2 = R^2 - 2hR + h^2 + r^2$

$\Rightarrow 2hR = h^2 + r^2$

$$\therefore R = \frac{h^2 + r^2}{2h}$$



Hence option (a) is correct.

Now from equation of surface

$$y = y_0 + \frac{\omega^2 r^2}{2g}$$

$$h = \frac{\omega^2 r^2}{2g}$$

Now using formula for refraction at curved surface

$$\frac{\mu_2}{v} - \frac{\mu_1}{u} = \frac{\mu_2 - \mu_1}{R}$$

$$\frac{1}{V} - \frac{\frac{4}{3}}{-(H-h)} = \frac{1-\frac{4}{3}}{R} \Rightarrow \frac{1}{V} - \frac{1}{3R} - \frac{4}{3H}$$

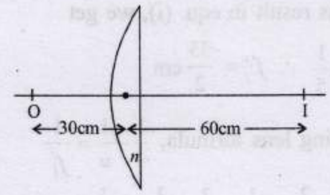
$[\because h << H]$

$$\Rightarrow \frac{1}{V} = \frac{2h}{3r^2} - \frac{4}{3H} \Rightarrow \frac{1}{V} = \frac{4}{3H}\left[1 - \frac{\omega^2 H}{4g}\right]$$

$$\therefore v = \frac{3H}{4}\left[1 + \frac{\omega^2 H}{4g}\right]^{-1} \quad \text{(Apparent depth)}$$

Hence option (d) is correct.

26. **(a, d) For lens, applying lens formula**



$$\frac{1}{v} - \frac{1}{u} = \frac{1}{f} \Rightarrow \frac{1}{v} - \frac{1}{-30} = \frac{1}{f}$$

$$\frac{1}{60} - \frac{1}{-30} = \frac{1}{f} \Rightarrow \frac{1}{f} = \frac{1}{60} + \frac{1}{30}$$

$\therefore$ $f = 20$ cm

$$\because \quad m = \frac{v}{u} \Rightarrow -2 = \frac{v}{-30} \Rightarrow v = 60 \text{ cm}$$

For reflection, applying mirror formula,

$$\frac{1}{v} + \frac{1}{u} = \frac{1}{f} = \frac{2}{R} \Rightarrow \frac{1}{10} + \frac{1}{-30} = \frac{2}{R} \Rightarrow R = 30\text{cm}$$

The image formed by convex side is faint erect and virtual.

By lens maker formula

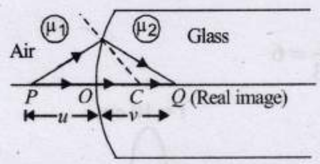
$$\frac{1}{f} = \left(\frac{n_l}{n_s} - 1\right)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$$

$$\therefore \frac{1}{20} = \left(\frac{n_l}{1} - 1\right)\left(\frac{1}{30}\right) \quad \therefore n_l = 2.5$$

(Refractive index of lens)

27. **(a)** Using formula for spherical refracting surface

$$\frac{-\mu_1}{u} + \frac{\mu_2}{v} = \frac{\mu_2 - \mu_1}{R}$$



Here $u = -x$, $v = +x$, $R = +R$, $\mu_1 = 1$, $\mu_2 = 1.5$

$$\frac{-1}{-x} + \frac{1.5}{x} = \frac{1.5-1}{R} \quad \therefore \; x = 5R = PO$$

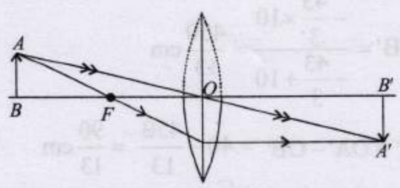
28. **(b)** Spherical aberation is smaller when the curved surface is facing the object because the total deviation is shared between the curved and the plane surfaces.

29. **(b, c)** Image formed by concave lens and convex mirror is virtual and erect. Concave lens and convex mirror are diverging in nature.

Therefore the refracted/reflected rays do not meet.

These rays are produced backwards to make them meet.

30. **(b, d)** The image formed will be complete because light rays from all parts of the object will strike on the lower half.



But since the upper half light rays are cut off, the intensity will reduce.

31. **(a)** Power of lens combination,

$$P = \frac{1}{f} = \frac{1}{f_1} + \frac{1}{f_2} = \frac{1}{0.4} + \frac{1}{-0.25} = -1.5 \text{ D}$$

32. **(a)** Total angle of deviation of the emergent ray with respect to the incident ray

$\alpha = (\theta_0 - \phi_0) + (\pi - 2\phi_0) + (\theta_0 - \phi_0)$

$\alpha = \pi + 2\theta_0 - 4\phi_0$ .... (i)

$\sin\theta_0 = n \sin\theta_0$....(ii)

For (P) $n = 2, \alpha = 180°$

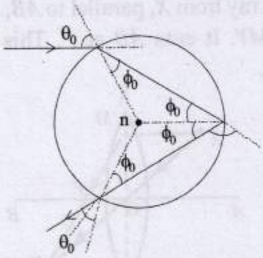
if $\alpha = \pi, 2\theta_0 - 4\phi_0 = 0$

$\theta_0 = 2\phi_0$

$\sin\theta_0 = 2 \sin\left(\frac{\theta_0}{2}\right)$

For (Q) $n = \sqrt{3}, \alpha = 180°$

$\theta_0 = 2\phi_0$



$\sin\theta_0 = \sqrt{3} \sin\left(\frac{\theta_0}{2}\right)$

or, $\theta_0 = 60°, 0°$

For (R) $n = \sqrt{3}, \alpha = 180°$

$\theta_0 = 2\phi_0$

$\sin\theta_0 = \sqrt{3}\sin\phi_0$

$\sin 2\theta_0 = \sqrt{3}\sin\phi_0$

$\cos\phi_0 = \sqrt{3}/2$

$\therefore \phi_0 = 30°, 0°$

For (s) $n = \sqrt{2}, \theta_0 = 45°$

$\sin 45° = \sqrt{2}\cos\phi_0$

$\cos\phi_0 = 1/2$ or $\phi_0 = 60°$

$\alpha = 180 + 2\theta_0 - 4\phi_0$

or, $\alpha - 180° + 90° - 120° = 150°$

33. **(a)** (i) $u_1 = -20$ cm

$f_1 = +10$ cm

$\frac{1}{v_1} = \frac{1}{f_1} + \frac{1}{u} \Rightarrow v = \frac{uf}{u+f}$

So, $v_1 = \frac{-20 \times 10}{-20 + 10} = 20$cm

Now, $u_2 = +15$ cm, $f_2 = +15$ cm

So, $v_2 = \frac{15 \times 15}{15 + 15} = 7.5$cm (from lens 2). So (I) → (P)

(ii) $u_1 = -20$ cm

$f_1 = +10$cm

So, $v_1 = \frac{-20 \times 10}{-20 + 10} = 20$cm

Now, $u_2 = +15$ cm, $f_2 = -10$ cm

So, $v_2 = \frac{15 \times -10}{15 - 10} = -30$cm. So (II) → (R)

(iii) Preceeding as above

$u_2 = +15$ cm, $f_2 = -20$ cm

So, $v_2 = \frac{15 \times -20}{15 - 20} = 60$ cm. So (III) → (Q)

(iv) $u_1 = -20$ cm, $f_1 = -20$ cm

$v_1 = \frac{-20 \times -20}{-20 - 20} = -10$cm

So, $u_2 = -15$ cm and $f_2 = +10$ cm

Then, $v_2 = \frac{-15 \times 10}{-15 + 10} = 30$cm. So (IV) → (T)

34. **(b)** For double convex lens, (P) $\frac{1}{f} = (\mu - 1)\left[\frac{1}{R_1} - \frac{1}{R_2}\right]$

$\Rightarrow (1.5 - 1)\left(\frac{1}{r} - \frac{1}{-r}\right) = (1.5 - 1)\left[\frac{2}{r}\right] = \frac{1}{r} \Rightarrow f = r$

$\frac{1}{F_{eq.}} = \frac{1}{f_1} + \frac{1}{f_2} = \frac{1}{r} + \frac{1}{r} = \frac{2}{r}$

$\therefore F_{eq} = \frac{r}{2}$

For (Q) plano-convex lens $\frac{1}{f} = (\mu - 1)\left[\frac{1}{R_1} - \frac{1}{R_2}\right]$

$= (1.5 - 1)\left[\frac{1}{\infty} - \frac{1}{-r}\right] = \frac{0.5}{r} = \frac{1}{2r} \quad \therefore f = 2r$

$\frac{1}{F_{eq.}} = \frac{1}{f_1} + \frac{1}{f_2} = \frac{1}{2r} + \frac{1}{2r} = \frac{2}{2r} = \frac{1}{r} \therefore F_{eq.} = r$

For (R) plano-concave lens

$\frac{1}{f} = (1.5 - 1)\left(\frac{1}{-r} - \frac{1}{\infty}\right) \Rightarrow f = -2r$

$\frac{1}{F_{eq.}} = \frac{1}{f} + \frac{1}{f} = \frac{1}{-2r} + \frac{1}{-2r} \Rightarrow F_{eq.} = -r$

For (S) combination of one double convex and one plano-concave lens

$\frac{1}{F_{eq.}} = \frac{1}{r} + \frac{1}{-2r} = \frac{1}{2r} \Rightarrow F_{eq.} = 2r$

35. **A-p, r; B-q,s,t; C-p,r,t, D-q,s**

When a ray of light enters from rarer medium to denser medium it bends towards the normal and its opposite when the ray of light travels.

From denser to rarer, it bends away from the normal.

(a) As $\mu_1 < \mu_2$, the ray of light entering the lens will bend towards the normal.

(B) As $\mu_1 > \mu_2$, the ray of light entering the lens will bend away from the normal.

(C) As $\mu_2 = \mu_3$, the ray of light coming out from the lens without any deviation.

(D) As $\mu_2 > \mu_3$, the ray of light coming out of the lens deviates away from the normal.

36. **A-p, q, r, s; B-q; C-p, q, r, s; D-p, q, r, s**

Image formed by concave mirror (A) convex lens (C) and concavo-convex lens (D)

Can be real, virtual, magnified, diminished. And image is at infinity when object is at focus.

Image formed by convex mirror (B) is always virtual.

37. Given: $f = 0.3$ m, $u = -0.4$ m

Using lens formula, $\frac{1}{v} - \frac{1}{-0.4} = \frac{1}{0.3} \Rightarrow v = 1.2$ m

Now, differentiating lens formula, $\frac{1}{v} - \frac{1}{u} = \frac{1}{f}$ w.r.t. time t

$$-\frac{1}{v^2}\frac{dv}{dt} + \frac{1}{u^2}\frac{du}{dt} = 0 \text{ given } \frac{du}{dt} = 0.01 \text{ m/s}$$

$$\therefore \left(\frac{dv}{dt}\right) = \frac{(1.20)^2}{(0.4)^2} \times 0.01 = 0.09 \text{ m/s}$$

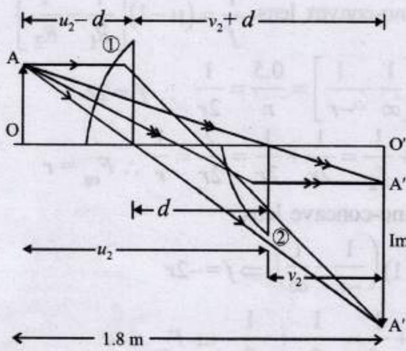
Therefore, magnitude of rate of change of position of the image (w.r.t. the lens) = 0.09 m/s

Lateral magnification, $m = \frac{v}{u}$ $\therefore \frac{dm}{dt} = \frac{u\frac{dv}{dt} - v\frac{du}{dt}}{u^2}$

$$= \frac{-(0.4)(0.09) - (1.2)(0.01)}{(0.4)^2} = -0.35 \text{ s}^{-1}$$

i.e., Magnitude of rate of change of lateral magnification $= 0.35 \text{ s}^{-1}$.

38. The first half lens (1) forms image of OA at O′A′ and the second half lens (2) forms image of OA at O′A′.



Here, $u_2 + v_2 = 1.8$ m ... (i)

The magnification of lens (1) is 2 (given)

$$\therefore \quad 2 = \frac{v_2 + d}{u_2 - d} \quad \text{...(ii)}$$

From eq. (i) and (ii)

$$u_2 = 0.6 + d, \; v_2 = 1.2 - d$$

Applying lens formula for lens (1)

$$\frac{1}{v_2 + d} + \frac{1}{u_2 - d} = \frac{1}{f} \quad \text{.... (iii)}$$

Again applying lens formula for lens (2)

$$\frac{1}{v_2} + \frac{1}{u_2} = \frac{1}{f} \quad \text{... (iv)}$$

From eq. (iii) and (iv)

$$\frac{1}{v_2 + d} + \frac{1}{u_2 - d} = \frac{1}{v_2} + \frac{1}{u_2}$$

$$\Rightarrow \frac{1}{1.2 - d + d} + \frac{1}{0.6 + d - d} = \frac{1}{1.2 - d} + \frac{1}{0.6 + d}$$

$\therefore$ $d = 0.6$ m

Substituting this value in eq. (iv) $\frac{1}{v_2} + \frac{1}{u_2} = \frac{1}{f}$

$$\Rightarrow \frac{1}{1.2 - d} + \frac{1}{0.6 + d} = \frac{1}{f}$$

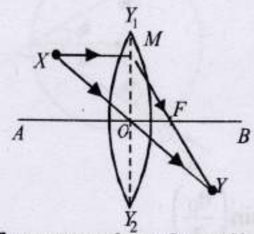
$$\frac{1}{1.2 - 0.6} + \frac{1}{0.6 + 0.6} = \frac{1}{f} \quad \therefore \quad f = 0.4 \text{ m}$$

39. (i) Since Y is below of optic axis AB and X above, therefore the image is real and inverted and lens should be a convex lens.

Steps of construction of ray diagram for a lens:

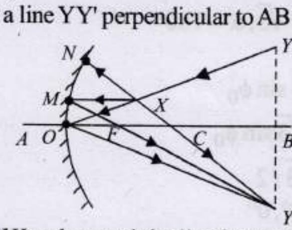
(1) Join XY. This represents the ray originating from the source X and meeting the image Y. Since the ray is undeviated after passing through the lens, therefore O is the optical centre of the lens. Draw Y_1OY_2 perpendicular to AB.

(2) Draw a ray from X, parallel to AB, XM. It strikes Y_1OY_2 at M. Join MY. It cuts AB at F. This is the focus of the convex lens.



(ii) **Steps of construction of ray diagram for concave mirror**

(1) Draw a line YY′ perpendicular to AB such that BY = BY′

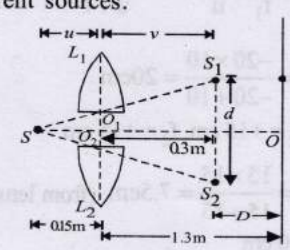


(2) Join $Y'X$ and extend the line it cuts the line AB at point O. The point O is the pole of mirror.

(3) Join X and Y. It cuts AB at the point C. This point C is the centre of curvature of the mirror.

(4) Draw XM parallel to the principal axis. Join M to Y. Let it cut AD at F. Therefore, F is the focus of concave mirror.

40. (i) Here, the two identical halves of convex lens L_1 and L_2 will form two seperate images S_1 and S_2 of the source S at equal distance. These images S_1 and S_2 will behave as two coherent sources.



For lens L_1 and L_2

$u = -0.15$ m, $v = ?$, $f = +0.1$ m

$$\frac{1}{v} - \frac{1}{u} = \frac{1}{f} \Rightarrow \frac{1}{v} = \frac{1}{f} + \frac{1}{u} = \frac{1}{0.1} + \frac{1}{-0.15}$$

$\therefore$ $v = 0.3$ m

Linear magnification, $m = \frac{v}{u}$

$\therefore \quad m = \frac{0.3}{-0.15} = -2$

$\therefore$ Two images S_1 and S_2 of S will be formed at 0.3 m from the lens.

S_1 will be 0.5 mm above its optic axis as $m = -2$.

S_2 will be 0.5 mm below its optic axis as $m = -2$.

$\therefore \quad d = S_1S_2 = 1.5\text{ mm} = 1.5 \times 10^{-3}\text{ m}$

$D = 1.30 - 0.30 = 1.0\text{ m}$

$\lambda = 500\text{ nm} = 500 \times 10^{-9}\text{ m} = 5 \times 10^{-7}\text{ m}$.

$\therefore$ Fringe width $\beta = \frac{D\lambda}{d}$

or $\beta = \frac{(1.0)(5\times10^{-7})}{1.5\times10^{-3}}$ or $\beta = \frac{1}{3} \times 10^{-3}$ m

Since point A is at third maxima,

$OA = 3\beta = 3 \times \left(\frac{1}{3}\times10^{-3}\right) = 10^{-3}$ m

$\therefore \quad OA = 10^{-3}$ m

(ii) Variation of OA with gap between L_1 and L_2.

If the gap between L_1 and L_2 is reduced, d will decrease.

$\because \quad \beta = \frac{D\lambda}{d} \quad \therefore$ β will increase.

$\therefore$ Distance OA will increase.

41. Ray of light first enters from medium II to medium III, i.e., from denser to rarer medium.

Case (i) : When $n_3 < n_1$

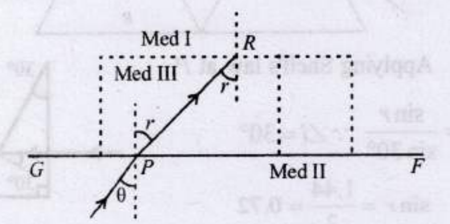
i.e., $n_3 < n_1 < n_2$

or, $\frac{n_3}{n_2} < \frac{n_1}{n_2}$ or, $\sin^{-1}\left(\frac{n_3}{n_2}\right) < \sin^{-1}\left(\frac{n_1}{n_2}\right)$

Obviously $n_3 < n_2$ and the angle θ is greater than the critical angle required for the ray passing from medium II to medium III. Therefore total internal reflection will also take place when a ray strikes with the same angle at the interface of medium II and medium III.

Case (ii) : $n_3 > n_1$ but $n_3 < n_2$

The ray will get refracted in medium III as the angle θ will now be less than the critical angle required for medium II and medium III pair.



Applying Snell's law at P

$$\frac{\sin\theta}{\sin r} = \frac{n_3}{n_2} \quad \therefore \sin r = \frac{n_2}{n_3}\sin\theta$$

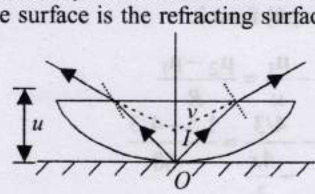
$\because \quad n_2 > n_3 \quad \therefore \quad r > \theta$

When the refracted ray PR meets the boundary DE, it is travelling from a denser medium to a rarer medium. Hence the ray will be totally internally reflected at DE if its angle of incidence r is more than the critical angle for med III and I.

$\sin i'' = \frac{n_1}{n_3} \quad \because \sin r > \frac{n_1}{n_3} \Rightarrow \sin r > \sin i'' \Rightarrow r > i''$

$\therefore$ Ray PR will be totally internal reflected along RQ. On reaching Q, the ray will be refracted in med II. Hence, the ray will ultimately be reflected back in medium II.

42. Here plane surface is the refracting surface $\therefore R = \infty$



$$-\frac{\mu_1}{u} + \frac{\mu_2}{v} = \frac{\mu_2 - \mu_1}{R} \Rightarrow -\frac{\mu_1}{-4} + \frac{\mu_2}{-3} = 0$$

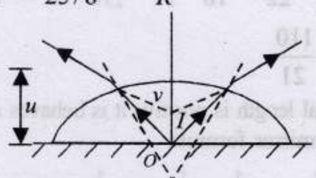
$$\therefore \quad \frac{\mu_2}{\mu_1} = \frac{3}{4} \qquad \text{...(i)}$$

Again refraction takes place from spherical surface,

$$-\frac{\mu_1}{u} + \frac{\mu_2}{v} = \frac{\mu_2 - \mu_1}{R}$$

$$\Rightarrow -\frac{1}{u} + \frac{\mu_2/\mu_1}{v} = \frac{(\mu_2/\mu_1) - 1}{R}$$

$$\Rightarrow -\frac{1}{-4} + \frac{3/4}{-25/8} = \frac{3/4 - 1}{R} \Rightarrow R = 25\text{ cm}$$



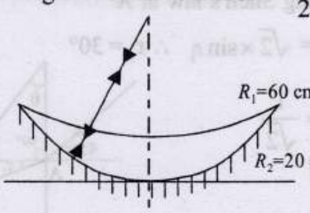
Now, applying Len's maker formula,

$$\frac{1}{f} = (\mu - 1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right) = \left(\frac{4}{3} - 1\right)\left(\frac{1}{25} - \frac{1}{\infty}\right) \therefore f = 75\text{cm}$$

43. (i) Here the silvered concavo-convex lens behaves like a mirror whose focal length $\frac{1}{f} = \frac{2}{f_1} + \frac{1}{f_2}$

f_1 = focal length of concave surface $= \frac{60}{2} = 30$ cm

f_2 = focal length of concave mirror $= \frac{20}{2} = 10$ cm



$$\therefore \quad \frac{1}{f} = \frac{2}{-30} + \frac{1}{-10} = \frac{4}{30} \Rightarrow f = -7.5\text{ cm}$$

Let P in be placed at a distance x on the optic axis such that its image is formed at the same place i.e., $u = v = x$

Using mirror formula

$$\frac{1}{f} = \frac{1}{v} + \frac{1}{u} \Rightarrow \frac{1}{-7.5} = \frac{1}{-x} + \frac{1}{-x}$$

$\therefore \quad x = 15$ cm

(ii) When concave part is filled with water of $\mu = \frac{4}{3}$.

Now, before striking with the concave surface, the ray is first refracted from a plane surface. So, let y be the distance of pin, then the plane surface will form its image at a distance $\frac{4}{3}y$ ($h_{app.} = \mu h$) from it.

Using $\frac{\mu_2}{v} - \frac{\mu_1}{u} = \frac{\mu_2 - \mu_1}{R}$

$$\frac{1.5}{-20} - \frac{4/3}{-\frac{4x}{3}} = \frac{1.5 - 4/3}{-60}$$

or $\frac{1}{x} = \frac{3}{40} - \frac{1}{360}$ or $x = 13.84$ cm

$\therefore \quad \Delta x = x_1 - x_2 = 15\text{ cm} - 13.84\text{ cm} = 1.16$ cm

44. From figure (i), there is no refraction. Therefore $\mu_1 = \mu$

From figure (ii), here the convex lens behaves as a diverging lens. Therefore, $\mu < \mu_2 \therefore \mu_1 < \mu_2$

45. Focal length of the equivalent mirror

$$\frac{1}{F} = \frac{1}{f_\ell} + \frac{1}{f_\ell} + \frac{1}{f_m} = \frac{2}{f} + \frac{1}{f_m}$$

$$= \frac{2}{20} + \frac{2}{22} = \frac{1}{10} + \frac{1}{11} = \frac{21}{110}$$

$$\Rightarrow F = \frac{110}{21}$$

As the focal length is positive it is behaves as concave mirror

Applying mirror formula

$$\frac{1}{u} + \frac{1}{v} = \frac{1}{f} \Rightarrow \frac{1}{-10} + \frac{1}{v} = \frac{1}{-110/21}$$

$$\Rightarrow \frac{1}{v} = \frac{1}{10} - \frac{21}{110}$$

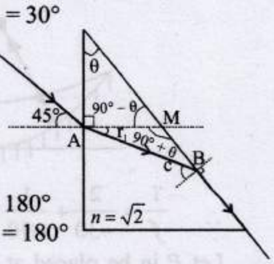
$v = -11$cm

The negative sign shows that the image formed infront of mirror hence the image is real.

Topic-4: Prism and Dispersion of Light

1. **(a)** Applying Snell's law at A

$1 \times \sin 45° = \sqrt{2} \times \sin r_1 \quad \therefore r_1 = 30°$



$\sin C = \frac{1}{n} = \frac{1}{\sqrt{2}}$

$\therefore \quad C = 45°$

In ΔAMB,

$90° + \theta + r_1 + (90° - C) = 180°$

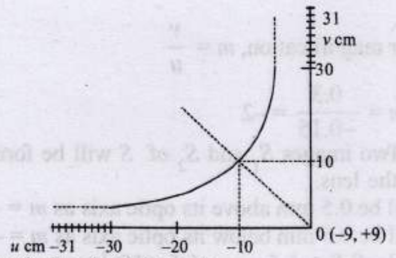
$\Rightarrow \quad 90° + \theta + 30° + 90° - 45° = 180°$

$\therefore \quad \theta = 15°$

2. **(c)** From the graph $u = -10$ cm; $v = 10$ cm $\Delta u = \Delta v = 0.1$

From lens formula, $\frac{1}{f} = \frac{1}{v} - \frac{1}{u} = \frac{1}{10} - \frac{1}{-10}$

$\therefore \quad f = 5$ cm



Differentiating lens formula,

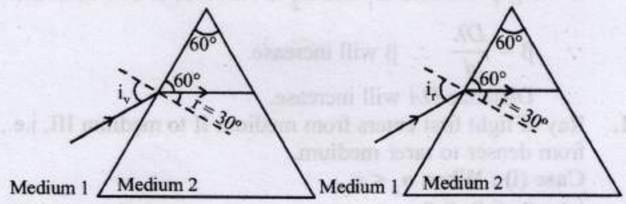
$$\frac{1}{f} = \frac{1}{v} - \frac{1}{u} \Rightarrow \frac{\Delta f}{f^2} = \frac{\Delta v}{v^2} + \frac{\Delta u}{u^2} \quad \text{(for maximum error in f)}$$

$$\Rightarrow \frac{\Delta f}{25} = \frac{0.1}{(10)^2} + \frac{0.1}{(10)^2}$$

$$\Rightarrow \Delta f = 25 \times 0.1 \times 2 \times 0.01 = 0.05$$

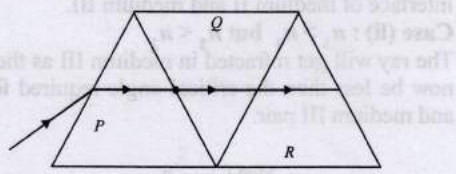
$\therefore \quad$ Focal length, $f \pm \Delta f = (5.00 \pm 0.05)$ cm.

3. **(a)** For minimum deviation the ray in the prism is parallel to the base of the prism. This condition does not depend on the colour (or wave length) of incident radiation. So in both the cases, for both the colours by geometry, angle of refraction, r = 30°.

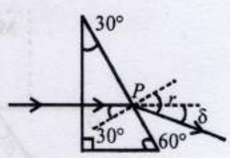


4. **(b)** For minimum deviation, incident angle is equal to emerging angle. And ray QR inside the equilateral prism is parallel to base.

5. **(c)** There will be no refraction from P to Q and then from Q to R all being identical made of same material. Hence the ray will now have the same deviation.



6. **(c)** Applying Snell's law at P,



$$\mu = \frac{\sin r}{\sin 30°} \quad \because \angle i = 30°$$

$$\Rightarrow \quad \sin r = \frac{1.44}{2} = 0.72$$

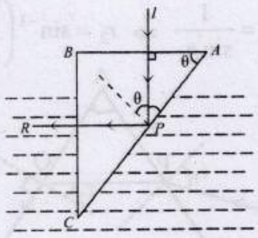
$\therefore \quad \delta = r - 30° = \sin^{-1}(0.72) - 30°$

$\therefore \quad$ The rays make an angle of

$2\delta = 2\,[\sin^{-1}(0.72) - 30°]$ with each other.

7. **(a)** The phenomenon of total internal reflection takes place during reflection at P to reach at BC

$$\therefore \sin\theta_{critical} = \frac{1}{{}_g^w\mu} = \frac{M_w}{M_g} = \frac{4/3}{3/2} = \frac{8}{9}$$



$\therefore$ $\sin\theta$ should be greater than critical angle $\theta_c = \frac{8}{9}$.

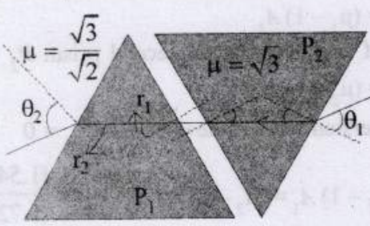
8. **(12)** By using optical reversibility principle
For prism P_2 : Minimum deviation using Snell's law,

$1 \times \sin\theta_1 = \sqrt{3}\sin r \quad r_1 = r_2 = \frac{A}{2} = \frac{60^\circ}{2} = 30^\circ$

$\sin\theta_1 = \sqrt{3} \times \frac{1}{2}$

$\therefore \quad i = e = 60^\circ$

For prism P_1 : Incident angle $= 60^\circ$



$1 \times \sin 60^\circ = \frac{\sqrt{3}}{\sqrt{2}}\sin r_1$

or, $\frac{\sqrt{3}}{2} = \frac{\sqrt{3}}{\sqrt{2}}\sin r_1$

$r_1 + r_2 = 60^\circ$

$\sin r_1 = \frac{1}{\sqrt{2}} \therefore r_1 = 45^\circ$ and $r_2 = 15^\circ$

$\frac{\sqrt{3}}{\sqrt{2}}\sin(45^\circ) = 1 \times \sin\theta$

$15^\circ = \frac{\pi \times 15}{180}\text{rad} = \frac{\pi}{12}\text{rad}$

$\theta = \sin^{-1}\left[\frac{\sqrt{3}}{\sqrt{2}}\sin\left(\frac{\pi}{12}\right)\right] \therefore \beta = 12$

9. **(1.50)**
In ΔXYZ, $90^\circ \times r + 90^\circ - C + 75^\circ = 180^\circ$

$\therefore \quad r + C = 75 \Rightarrow r = 75^\circ - C$ ....(i)

Applying Snell's law at Z

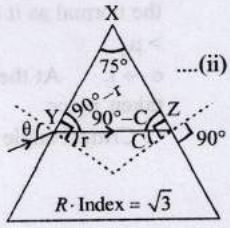
$\sqrt{3}\ \sin C = n \sin 90^\circ$

$\sqrt{3}\ \sin C = n$ ....(ii)

Applying snell's law at Y

$1 \times \sin\theta = \sqrt{3}\ \sin r$

$= \sqrt{3}\ \sin(75^\circ - C)$



From eq. (i)
For $\theta = 60^\circ$

$\frac{\sqrt{3}}{2} = \sqrt{3}\sin(75^\circ - C) \quad \therefore \quad C = 45^\circ$

From eq. (ii) $n = \sqrt{3}\sin 45^\circ = \frac{\sqrt{3}}{\sqrt{2}}$

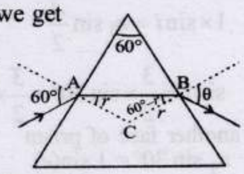
$\therefore \quad n^2 = 1.50$

10. **(2)** Applying Snell's law at A

$\sin 60^\circ = n \sin r$ ...(i)

Differentiating w.r.t 'n' we get

$0 = \sin r + n\cos r \times \frac{dr}{dn}$ ...(ii)



Again, applying Snell's law at B

$\sin\theta = n\sin(60^\circ - r)$ ...(iii)

Differentiating w.r.t 'n' we get

$$\cos\theta\frac{d\theta}{dn} = \sin(60^\circ - r) + n\cos(60^\circ - r)\left[-\frac{dr}{dn}\right]$$

$$\therefore \quad \cos\theta\frac{d\theta}{dn} = \sin(60^\circ - r) - n\cos(60^\circ - r)\left[-\frac{\tan r}{n}\right]$$

[from (ii)]

$$\therefore \quad \frac{d\theta}{dn} = \frac{1}{\cos\theta}[\sin(60^\circ - r) + \cos(60^\circ - r)\tan r] \quad ...(iv)$$

From eq. (i), for $n = \sqrt{3}$ we get $r = 30^\circ$

From eq (iii), for $n = \sqrt{3}$, $r = 30^\circ$ we get $\theta = 60^\circ$

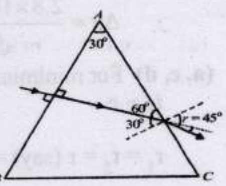
Substituting the values of r and θ in eq (iv) we get

$$\frac{d\theta}{dn} = \frac{1}{\cos 60^\circ}[\sin 30^\circ + \cos 30^\circ \tan 30^\circ] = 2\left(\frac{1}{2} + \frac{1}{2}\right) = 2$$

11. At face AB of prism ray falls normally using Snell's law for the refraction at AC,

$\mu\sin i = (1)\sin r$

$\sqrt{2}\sin 30^\circ = \sin r \Rightarrow r = 45^\circ$



Angle of deviation at face AC

$= 45^\circ - 30^\circ = 15^\circ$

12. $\mu = \frac{\sin\left(\frac{A + \delta m}{2}\right)}{\sin A/2}$, $\therefore \sqrt{2} = \frac{\sin\left(\frac{60 + \delta m}{2}\right)}{\sin 60/2}$

($\angle A = 60^\circ$ for an equilateral triangle)

$\Rightarrow \frac{60 + \delta m}{2} = 45^\circ \Rightarrow \delta m = 30^\circ$

The condition is for minium deviation. In this case the ray inside the prism becomes parallel to base. Therefore the angle made by the ray inside the prism with the base of the prism is 0°.

13. **True**

For the light to split, the material through which the light passes should have refractive index greater than 1.

Since the prism is hollow, we get no spectrum as no refraction and no dispersion occur. The thickness of glass slabs through which the prism is made can be neglected.

14. **(b, c)** Given angle of prism $A = 60°$

For minimum deviation.

$$r_1 = r_2 = A/2 = 30°$$

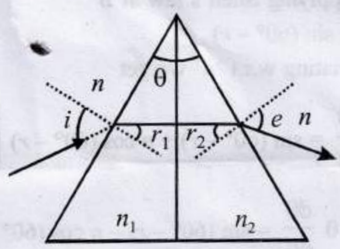
and from Snell's law, $n \times \sin i = n_1 \sin r_1$

$$\Rightarrow \quad 1 \times \sin i = n_1 \sin \frac{A}{2}$$

$$\therefore \quad \sin i = \frac{3}{2} \times \sin 30° = \frac{3}{2} \times \frac{1}{2} \quad \Rightarrow \sin i = \frac{3}{4}$$

At another face of prism

$$n_1 \sin 30° = 1 \sin(e)$$



On differentiating both sides $\Delta n \sin 30° = \Delta e \cos (e)$

$$\Delta e = \frac{\Delta n}{2\cos(e)}$$

$$\text{or,} \quad \Delta e = \frac{\Delta n}{2\sqrt{1-\frac{9}{16}}} = \frac{2}{\sqrt{7}} \Delta n$$

$$\Delta e = \frac{2}{\sqrt{7}} \Delta n \Rightarrow \Delta e < \Delta n \text{ and } \Delta e \propto \Delta n$$

$$\Delta n = 2.8 \times 10^{-3}$$

$$\therefore \quad \Delta e = \frac{2.8 \times 10^{-3} \times 2}{\sqrt{7}} = 2.11 \times 10^{-3} \text{ rad} = 2.11 \text{ mrad}$$

15. **(a, c, d)** For minimum deviation

$$i_1 = e$$

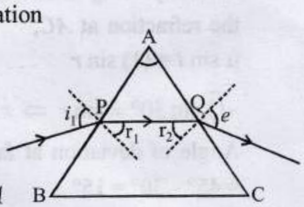
$$r_1 = r_2 = r \text{ (say)} = \frac{A}{2}$$

$$\delta_m = 2i_1 - A,$$

Here $\delta_m = A \Rightarrow 2i_1 - A = A$

$$\therefore \quad i_1 = A \Rightarrow e = A$$

$$\therefore \quad r_1 = \frac{i_1}{2}$$

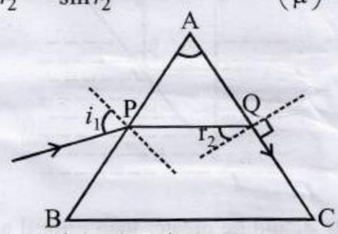


Relation between μ and A

$$\mu = \frac{\sin i_1}{\sin r_1} = \frac{\sin A}{\sin A/2} = \frac{2\sin A/2 \cos A/2}{\sin A/2} = 2\cos A/2$$

When emergent ray tangential to the surface

$$\mu = \frac{\sin 90°}{\sin r_2} = \frac{1}{\sin r_2} \Rightarrow r_2 = \sin^{-1}\left(\frac{1}{\mu}\right)$$



But $r_1 + r_2 = A \quad \therefore \quad r_1 = A - r_2$

$$\therefore \quad r_1 = A - \sin^{-1}\left(\frac{1}{\mu}\right)$$

Applying Snell's law at 'P'

$$\mu = \frac{\sin i_1}{\sin r_1} \quad \therefore \quad i_1 = \sin^{-1}\left[\mu \sin\left(A - \sin^{-1}\frac{1}{\mu}\right)\right]$$

For minimum deviation through isoscletes prism if $\angle B = \angle C$, PQ || BC.

16. **(c)** Angle of deviation for the first prism P_1

$$\delta_1 = (\mu_1 - 1) A_1$$

Angle of deviation for the second prism P_2

$$\delta_2 = (\mu_2 - 1) A_2$$

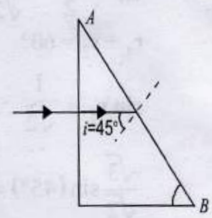
For dispersion without deviation, $\delta_{net} = 0$

$$\therefore \quad (\mu_1 - 1) A_1 = (\mu_2 - 1) A_2 \quad \text{or,} \quad A_2 = \frac{(1.54-1)}{(1.72-1)} 4° = 3°$$

17. **(a)** The colour beam of light for which $i > c$ will get total internal reflection.

$$\mu = \frac{1}{\sin C} = \frac{1}{\sin 45°} = 1.414$$

i.e. For an angle of incidence of 45°, that colour will suffer total internal reflection for which $\mu > 1.414$.



Therefore, red light for which $\mu <$ 1.414 will be refracted at interface AB whereas blue and green light will suffer total internal reflection.

18. **(d)** e → f. When the ray enters from the rectangular block to prism then angle of incidence > angle of refraction, so $\mu_2 > \mu_1$. The ray then moves away from the normal when it emerges out of the rectangular block. Therefore $\mu_2 > \mu_3$.

e → g. As there is no deviation of the ray as it emerges out of the prism, $\therefore \mu_2 = \mu_1$.

e → h. As the ray emerges out of prism, it moves away from the normal. $\therefore \mu_2 < \mu_1$. And the ray moves away from the normal as it emerges out of the rectangular block, $\therefore \mu_2 > \mu_3$.

e → i. At the prism surface, total internal reflection has taken place.

∴ Critical angle $45° > C \Rightarrow \sin 45° > \sin C$

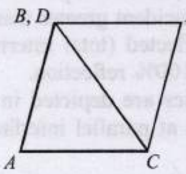
$$\Rightarrow \frac{1}{\sqrt{2}} > \frac{\mu_2}{\mu_1} \quad \therefore \mu_1 > \sqrt{2}\mu_2$$

19. (a) At minimum deviation, $r_1 = r_2 = \frac{A}{2} = 30°$

By Snell's law

$$\mu = \frac{\sin i_1}{\sin r_1} \Rightarrow \sqrt{3} = \frac{\sin i_1}{\sin 30°} \Rightarrow \sin i_1 = \frac{\sqrt{3}}{2} \Rightarrow i_1 = 60°$$

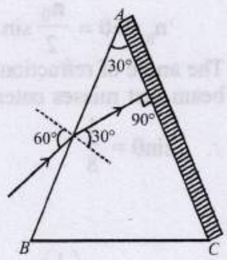
(b) In the position shown,
net deviation suffered by the ray of light should be minimum. Therefore, the second prism should be rotated by 60° (ACW)



20. (a) $\sin i_1 = \mu \sin r_1$

or $\sin 60° = \sqrt{3} \sin r_1$

$\therefore \quad \sin r_1 = \frac{1}{2}$ or $r_1 = 30°$



Now, $r_1 + r_2 = A$

$\therefore \quad r_2 = A - r_1 = 30° - 30° = 0°$

Therefore, ray of light falls normally on the face AC and angle of emergence $i_2 = 0°$

(b) Multiple reflection occur between surface of film. Intensity will be maximum if constructive interference take place in transmitted wave.

For maximum thickness, $\Delta x = 2\mu t = \lambda$

$$t = \frac{\lambda}{2\mu} = \frac{6600}{2 \times 2.2} = 1500 \text{ Å}$$

21. (a) The rays incident at any angle on the interface BC pass through without bending at this surface if $n_1 = n_2$

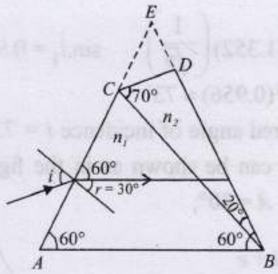
$$1.20 + \frac{10.8 \times 10^{-4}}{\lambda_0^2} = 1.45 + \frac{1.80 \times 10^{-4}}{\lambda_0^2}$$

(where λ_0 is in nm)

or $\lambda_0 = \left(\frac{9.0 \times 10^4}{0.25}\right)^{1/2} = 600$ nm

(b) For light of wavelength $\lambda_0 = 600$ nm, the combination of prism acts as a single prism shaped like an isosceles triangle *ABE* as shown in figure. At the minimum deviation, the ray inside the prism will be parallel to the base.

$$\therefore \quad r = \frac{A}{2} = \frac{60°}{2} = 30°$$



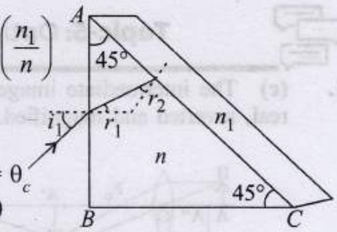
$$n_1 = \frac{\sin i}{\sin r} \Rightarrow \sin i = n_1 \sin r$$

$$\Rightarrow \sin i = \left[1.20 + \frac{10.8 \times 10^4}{(600)^2}\right] \times \frac{1}{2} \quad (\because \lambda_0 = 600 \text{ nm})$$

or, $\sin i = \left(\frac{3}{4}\right)$

$\therefore$ Angle of incidence, $i = \sin^{-1}\left(\frac{3}{4}\right)$

22. (i) At face *AC*



Critical angle, $\theta_c = \sin^{-1}\left(\frac{n_1}{n}\right)$

or $\sin \theta_c = \frac{n_1}{n}$

Now, it is given that $r_2 = \theta_c$

$\therefore r_1 = A - r_2 = (45° - \theta_c)$

Applying Snell's law at fact *AB*,

$$n = \frac{\sin i_1}{\sin r_1}$$

or $\sin i_1 = n \sin r_1$

$\therefore \quad i_1 = \sin^{-1}(n \sin r_1)$

Substituting value of r_1, we get

$i_1 = \sin^{-1}\{n \sin (45° - \theta_c)\}$

$= \sin^{-1}\{n(\sin 45° \cos \theta_c - \cos 45° \sin \theta_c)\}$

$$= \sin^{-1}\left\{\frac{n}{\sqrt{2}}(\sqrt{1-\sin^2\theta_c} - \sin\theta_c)\right\} = \sin^{-1}\left\{\frac{n}{\sqrt{2}}\left(\sqrt{1-\frac{n_1^2}{n^2}} - \frac{n_1}{n}\right)\right\}$$

This is the required angle of incidence (i_1) at face *AB* for which the ray strikes at the diagonal face *AC* at critical angle is

$$i_1 = \sin^{-1}\left\{\frac{1}{\sqrt{2}}(\sqrt{n^2 - n_1^2} - n_1)\right\}$$

(ii) Here $n_1 \neq n$ (because $n_1 < n$ is given) so for ray to pass undeviated through *AC*

$r_2 = 0°$ (Ray ⊥ on face *AC*)

or $r_1 = A - r_2 = 45° - 0° = 45°$

Now applying Snell's law at face *AB*, we have $n = \frac{\sin i_1}{\sin r_1}$

or $1.352 = \frac{\sin i_1}{\sin 45°}$

$\therefore \quad \sin i_1 = (1.352)\left(\frac{1}{\sqrt{2}}\right) \qquad \sin i_1 = 0.956$

$\therefore \quad i_1 = \sin^{-1}(0.956) \approx 73°$

i.e., the required angle of incidence $i = 73°$

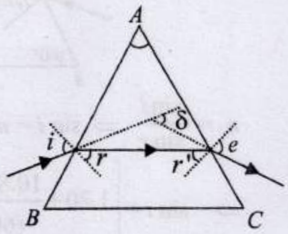
23. The situation can be shown as in the figure.

Here, $i = 60°$, $A = 30°$, $\delta = 30°, e = ?$

And, $A + \delta = i + e$

$\Rightarrow \quad e = A + \delta - i$

$= 30° + 30° - 60° = 0$



Hence the emergent ray is normal to the face from which it emerges.

When $e = 0$, $r' = 0$

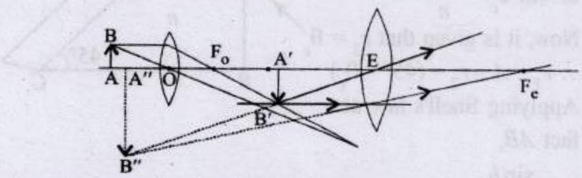
$r' + r = A = r = 30°$.

From Snell's law, refractive index of prism,

$$\mu = \frac{\sin i}{\sin r} = \frac{\sin 60°}{\sin 30°} = \frac{\sqrt{3}/2}{1/2} = \sqrt{3} = 1.732.$$

Topic-5: Optical Instruments

1. **(c)** The intermediate image in compound microscope is real, inverted and magnified.



2. **(a)** Here $f_0 = 2$ cm and $f_e = 3$ cm.

Using lens formula for eye piece

$$\Rightarrow \quad \frac{-1}{u_e} + \frac{1}{\infty} = \frac{1}{3} \Rightarrow u_e = -3 \text{ cm}$$

[$\because$ final image formed by the eyepiece at ∞]

Given distance between objective and eye piece = 15 cm

$\therefore$ Distance of image formed by the objective

$V_0 = 15 - 3 = 12$ cm.

Let u_0 be the object distance from objective, then for objective lens

$$-\frac{1}{u_0} + \frac{1}{v_0} = \frac{1}{f_0} \text{ or } \frac{-1}{u_o} + \frac{1}{12} = \frac{1}{2}$$

$$\Rightarrow \frac{-1}{u_o} = \frac{1}{2} - \frac{1}{12} = \frac{5}{12} \quad \therefore u_o = -\frac{12}{5} = -2.4 \text{ cm}$$

3. The resolving power of a microscope

$$\propto \frac{1}{\lambda \text{ used}}$$

The resolving power of an electron microscope is higher than that of an optical microscope because the wavelength of electrons is smaller than the wavelength of visible light.

4. **(a, b, c, d)** In an astronomical telescope the distance between the objective lens and eyepiece lens

$L = f_0 + f_e = 16 + 0.02 = 16.02$ m

$$\text{Angular magnification} = -\frac{f_{\text{objective}}}{f_{\text{eye piece}}} = \frac{-16}{0.02} = -800$$

The image seen by the astronomical telescope is inverted. Also the objective is larger than eye piece.

5. **(d)** In an astronomical telescope when the final image is formed at infinity:

Angular magnification

$$\therefore \quad M = \frac{f_o}{f_e} = 5 \Rightarrow f_o = 5f_e$$

And the separation between the objectives and eye-piece.

$L = f_o + f_e = 36 \Rightarrow 5f_e + f_e = 36$ or $f_e = 6$ cm

$\therefore \quad f_o = 5f_e = 5 \times 6 = 30$ cm

Topic-6: Miscellaneous (Mixed Concepts) Problems

1. **(c)** When the light is incident on glass - an interface travelling from glass at an angle less than critical angle a small part of light will be reflected and most part will be transmitted.

When the light is incident greater than the critical angle, it gets completed reflected (total internal reflection) so 0% transmission and 100% reflection.

These characteristics are depicted in option (c).

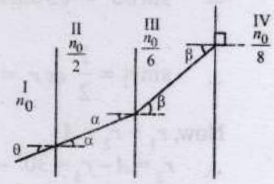
2. **(b)** For refraction at parallel interfaces

From Snell's law,

$$n_0 \sin\theta = \frac{n_0}{2} \sin\alpha = \frac{n_0}{6} \sin\beta = \frac{n_0}{8} \sin 90°$$

The angle of refraction in the region IV must be 90° as the beam just misses entering the region IV.

$$\therefore \quad \sin\theta = \frac{1}{8}$$



or, $\theta = \sin^{-1}\left(\frac{1}{8}\right)$

3. **(c)** The image I' for first refraction (i.e., when the ray comes out of liquid) is at a depth of

$$= \frac{33.25}{1.33} = 25 \text{ cm} \quad \left[\because \text{Apparent depth} = \frac{\text{Real depth}}{\mu}\right]$$

Now, reflection will occur at concave mirror. For this I' behaves as an object. Distance of object from mirror, $u = -(15 + 25) = -40$ cm

and $v = -\left[15 + \frac{25}{1.33}\right]$

Where $\frac{25}{1.33}$ is the real depth of the image.

Using mirror formula

$$\frac{1}{f} = \frac{1}{v} + \frac{1}{u} \Rightarrow \frac{1}{f} = \frac{1}{-33.8} + \frac{1}{-40}$$

$\therefore \quad f = -18.31$ cm

4. **(a)** Distance of virtual image from surface = 6 cm

The rays coming from the point object fall on the glass-air interface normally and hence pass undeviated. Therefore if we retrace the path of the refracted rays backwards, the image will be formed at the centre only.

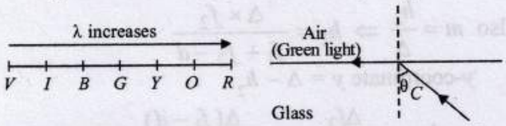
5. **(c)** Frequency does not change with change of medium.

$\therefore$ Frequency of sound in water = Frequency heard in air = 600 Hz

6. **(c)** $\sin\theta_C = \frac{1}{\mu}$ and $\mu \propto \frac{1}{\lambda}$

$\therefore \sin\theta_C \propto \lambda$

For higher value of λ, the critical angle θ_C also increases



Hence yellow, orange and red colours of light for which $\theta_C >$ incidence angle will come out to air.

7. **(c)** Since both surfaces have same radius of curvature $R_1 = R_2 = R$ on the same side, no dispersion will occur.

For no dispersim $\frac{1}{f} = (\mu - 1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right) = 0$

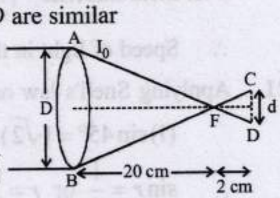
8. **(130.00)**

Let D be the initial area covered by light and d be the final area covered by light at 22 cm.

From figure, ΔAFB and ΔCFD are similar

$\therefore \frac{d}{D} = \frac{2}{20} = \frac{1}{10}$

$\therefore$ Ratio of area $= \frac{d^2}{D^2} = \frac{1}{100}$



As there is no energy loss $\therefore I_0 D^2 = I d^2$

$\therefore$ Average intensity of light at a distance 22 cm

$I = \frac{D^2}{d^2} I_0 = 1.3 \times 100 = 130.00\ \text{kWm}^{-2}$

9. Velocity of light in vacuum $c = \frac{1}{\sqrt{\mu_0 \varepsilon_0}}$

Velocity of light in a medium $v = \frac{1}{\sqrt{\mu\varepsilon}}$

Refractive index, $\mu = \frac{c}{v} = \frac{1/\sqrt{\mu_0\varepsilon_0}}{1/\sqrt{\mu\varepsilon}} = \frac{\sqrt{\mu\varepsilon}}{\sqrt{\mu_0\varepsilon_0}}$

10. For refraction at *APB*

$-\frac{\mu_2}{u} + \frac{\mu_1}{v} = \frac{\mu_1 - \mu_2}{R}$

$\Rightarrow \frac{-2}{-15} + \frac{1}{v} = \frac{1-2}{-10} \Rightarrow v = -30$ cm

Therefor the image of *O* will be formed at 30 cm to the right of *P*.

11. **True;** Intensity of a line source of power (*P*) at a distance *r* from the source

$I = \frac{P}{A} = \frac{P}{2\pi r l}$ or, $I \propto \frac{1}{r}$

12. **True;** This is due to atmospheric refraction. The light coming from sun bends towards the normal. As we go higher in the atmosphere density by decreases. Therefore, sun appears higher.

13. **(b, d)** As we know, $\mu = \frac{\text{real depth}}{\text{apparent depth}}$

Case - I $H_1 = \frac{H}{1.5} = \frac{30}{1.5} = 20\text{cm}$

Case - II $\frac{-n_1}{u} + \frac{n_2}{v} = \frac{n_2 - n_1}{R}$

$\therefore \frac{-1.5}{-30} + \frac{1}{-H_2} = \frac{1-1.5}{-300}$ $\therefore H_2 = 20.68$ cm

Case - III $\therefore \frac{-1.5}{-30} + \frac{1}{-H_3} = \frac{1-1.5}{300}$

$\therefore \frac{1}{-H_3} = -\frac{1}{600} - \frac{1}{20}$ $\therefore H_3 = 19.35$ cm

14. **(a, b, c)** $\frac{1}{f} = (n-1)\frac{2}{R} \Rightarrow f = \frac{R}{2(n-1)}$

$\frac{1}{f + \Delta f} = \frac{(n-1)}{R} + \frac{(n + \Delta n - 1)}{R} = \frac{2(n-1) + \Delta n}{R}$

$\therefore f + \Delta f = \frac{R}{2(n-1) + \Delta n}$

$\therefore \frac{f + \Delta f}{f} = \frac{R}{2(n-1) + \Delta n} \times \frac{[2(n-1)]}{R} = \frac{[2(n-1)]}{[2(n-1) + \Delta n]}$

$\therefore \frac{\Delta f}{f} = \frac{2n - 2 - 2n + 2 - \Delta n}{[2(n-1) + \Delta n]}$

$= \frac{-\Delta n}{2n - 2 + \Delta n} = \frac{-\Delta n}{2(n-1)}$ ...(i)

$[\because \Delta n << (n-1)]$

From equation (i) if $\frac{\Delta n}{n} < 0$, then $\frac{\Delta f}{f} > 0$.

Also, $\left|\frac{\Delta f}{f}\right| > \left|\frac{\Delta n}{n}\right|$

The relation between $\frac{\Delta f}{f}$ and $\frac{\Delta n}{n}$ remains unchanged, if convex surface are replaced by concave surface of the same radius of curvature.

For $n = 1.5$, $\Delta n = 10^{-3}$ and $f = 20$ cm then eq. (i)

$\frac{\Delta f}{20} = -\frac{10^{-3}}{2(1.5-1)} \Rightarrow \Delta f = -0.02$ cm

or $|\Delta f| = 0.02$ cm

15. **(b)** For refraction in S_1

$-\frac{n_1}{u} + \frac{n_2}{v} = \frac{n_2 - n_1}{R} \Rightarrow -\frac{1.5}{-50} + \frac{1}{V} = \frac{1-1.5}{-10}$

$\Rightarrow v = 50$ cm.

Again for refraction in S_2

$-\frac{n_1}{u} + \frac{n_2}{v} = \frac{n_2 - n_1}{R}$

$-\frac{1}{-(d-50)} + \frac{1.5}{\infty} = \frac{1.5-1}{10}$

$\therefore \quad \dfrac{1}{d-50} = \dfrac{1}{20} \Rightarrow d - 50 = 20 \therefore d = 70$ cm.

16. **(a, b, c)**

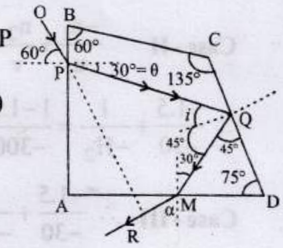
(a) Applying Snell's law at P

$n_1 \sin i = n_2 \sin r$

$\Rightarrow \quad n_1 \sin 60° = n_2 \sin\theta$

$\Rightarrow \quad \sin 60° = \sqrt{3}\sin\theta$

$\Rightarrow \quad \theta = 30° = r$



In quadrilateral BCQP,

$60° + (90° + 30°) + 135° + \angle PQC = 360°$

$\Rightarrow \angle PQC = 45° \quad \Rightarrow i = 45°$

The critical angle for prism - air pair of media

$C = \sin^{-1}\left(\dfrac{1}{\sqrt{3}}\right)$ which is less than 45°.

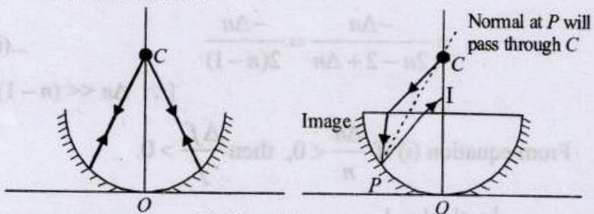
Therefore total internal reflection takes place at face CD.

(b) In ΔQDM, $\angle QMD = 180° - (45° + 75°) = 60°$

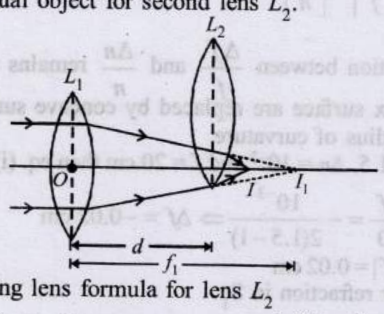
Therefore the angle of incidence of ray QM on AD is 30°. This angle is less than the critical angle. Hence the ray emerges out of face AD.

(c) From the figure, angle between the incident ray OP and the emergent ray MR is 90°.

17. **(d)** The ray diagram is shown in figure. Therefore, the image will be real and between C and O. When the mirror is filled with water.



18. **(c)** The image I_1 of parallel rays formed by lens L_1 will act as virtual object for second lens L_2.



Applying lens formula for lens L_2

$\Rightarrow \quad \dfrac{1}{v_2} - \dfrac{1}{f_1 - d} = \dfrac{1}{f_2} \Rightarrow v_2 = \dfrac{f_2(f_1-d)}{f_2 + f_1 - d}$

$\therefore$ Horizontal distance of the image I from O

$x = d + \dfrac{f_2(f_1-d)}{f_2+f_1-d} = \dfrac{f_1 f_2 + d(f_1-d)}{f_1+f_2-d}$

To find the y-coordinate, we use magnification formula for lens L_2

$$m = \frac{v_2}{u_2} = \frac{\dfrac{f_2(f_1-d)}{f_1+f_2-d}}{f_1-d} = \frac{f_2}{f_1+f_2-d}$$

Also $m = \dfrac{h_2}{\Delta} \Rightarrow h_2 = \dfrac{\Delta \times f_2}{f_1+f_2-d}$

$\therefore$ y-coordinate $y = \Delta - h_2$

$= \Delta - \dfrac{\Delta f_2}{f_1+f_2-d} = \dfrac{\Delta(f_1-d)}{f_1+f_2-d}$

19. **(c)** Let n_1 = refractive index of air and n_2 = refractive index of meta material which is negative

From Snell's law, $\dfrac{n_2}{n_1} = \dfrac{\sin\theta_1}{\sin\theta_2}$

$\because$ n_2 negative $\therefore$ θ_2 is also negative hence graph (c) is correct.

20. **(b)** Speed of light in a medium, $V = \dfrac{C}{n}$

n for meta-material $= |n|$

$\therefore$ Speed of light in the meta-material $V = \dfrac{C}{|n|}$.

21. Applying Snell's law on face AB,

$(1)\sin 45° = (\sqrt{2})\sin r$

$\therefore \quad \sin r = \dfrac{1}{2}$ or $r = 30°$

i.e., ray becomes parallel to AD inside the block.

Now applying,

$\dfrac{\mu_2}{v} - \dfrac{\mu_1}{u} = \dfrac{\mu_2 - \mu_1}{R}$ on face CD,

$\dfrac{1.514}{OE} - \dfrac{\sqrt{2}}{\infty} = \dfrac{1.514 - \sqrt{2}}{0.4}$

Solving this equation, we get OP = 6.06 m

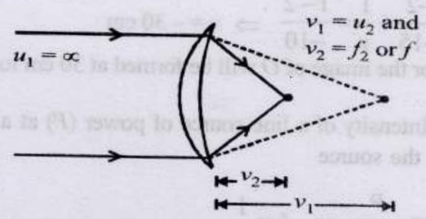
22. For refraction at first surface,

$\dfrac{\mu_2}{v_1} - \dfrac{\mu_1}{-\infty} = \dfrac{\mu_2 - \mu_1}{+R}$ ..... (i)

For refraction at second surface,

$\dfrac{\mu_3}{v_2} - \dfrac{\mu_2}{v_1} = \dfrac{\mu_3 - \mu_2}{+R}$ ..... (ii)

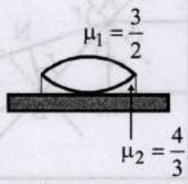
Adding Eqs. (i) and (ii), we get



$\dfrac{\mu_3}{v_2} = \dfrac{\mu_3 - \mu_1}{R}$ or $v_2 = \dfrac{\mu_3 R}{\mu_3 - \mu_1}$

Therefore, focal length of the given lens system is $\dfrac{\mu_3 R}{\mu_3 - \mu_1}$.

23. Let R be the radius of curvature of both the surfaces of the equi-convex lens. In the first case :



Let f_1 be the focal length of equi-convex lens of refractive index μ_1 and f_2 the focal length of plano-concave lens of refractive index μ_2. The focal length of the combined lens system will be given by

$$\frac{1}{F} = \frac{1}{f_1} + \frac{1}{f_2}$$

$$= (\mu_1 - 1)\left(\frac{1}{R} - \frac{1}{-R}\right) + (\mu_2 - 1)\left(\frac{1}{-R} - \frac{1}{\infty}\right)$$

$$= \left(\frac{3}{2} - 1\right)\left(\frac{2}{R}\right) + \left(\frac{4}{3} - 1\right)\left(-\frac{1}{R}\right)$$

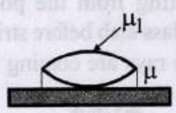
$$= \frac{1}{R} - \frac{1}{3R} = \frac{2}{3R} \text{ or } F = \frac{3R}{2}$$

Now, image coincides with the object when ray of light retraces its path or it falls normally on the plane mirror. This is possible only when object is at focus of the lens system.

Hence, F = 15 cm

or $\frac{3R}{2} = 15$ cm or $R = 10$ cm

In the second case, let μ be the refractive index of the liquid filled between lens and mirror and let F' be the focal length of new lens system. Then,



$$\frac{1}{F'} = (\mu_1 - 1)\left(\frac{1}{R} - \frac{1}{-R}\right) + (\mu - 1)\left(\frac{1}{-R} - \frac{1}{\infty}\right)$$

or $\frac{1}{F'} = \left(\frac{3}{2} - 1\right)\left(\frac{2}{R}\right) - \frac{(\mu - 1)}{R}$ or $= \frac{1}{R} - \frac{\mu - 1}{R} = \frac{(2 - \mu)}{R}$

$\therefore \quad F' = \frac{R}{2 - \mu} = \frac{10}{2 - \mu} \quad (\because R = 10 \text{ cm})$

Now, the image coincides with object when it is placed at 25 cm distance.

Hence, $F = 25$

or $\frac{10}{2 - \mu} = 25$

or $50 - 25\mu = 10$

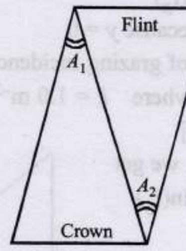
or $25\mu = 40$

$\therefore \quad \mu = \frac{40}{25} = 1.6$

or $\mu = 1.6$

24. (a) When angle of prism is small and angle of incidence is also small, the deviation is given by $\delta = (\mu - 1)A$.

Net deviation by the two prisms is zero. So,



or $(\mu_1 - 1)A_1 + (\mu_2 - 1)A_2 = 0$ ..... (i)

Here, μ_1 and μ_2 are the refractive indices for crown and flint glasses respectively.

Hence, $\mu_1 = \frac{1.51 + 1.49}{2} = 1.5$

and $\mu_2 = \frac{1.77 + 1.73}{2} = 1.75$

A_1 = Angle of prism for crown glass = 6°

Substituting the values in Eq. (i), we get

$$(1.5 - 1)(6°) + (1.75 - 1)A_2 = 0$$

This gives $A_2 = -4°$

Hence, angle of flint glass prism is 4° (Negative sign shows that flint glass prism is inverted with respect to the crown glass prism.)

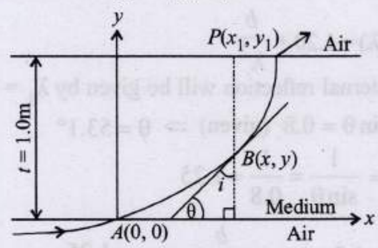
(b) Net dispersion due to the two prisms is

$$= (\mu_{b_1} - \mu_{r_1})A_1 + (\mu_{b_2} - \mu_{r_2})A_2$$

$$= (1.51 - 1.49)(6°) + (1.77 - 1.73)(-4°) = -0.04°$$

$\therefore$ Net dispersion is $-0.04°$

25. (a) A ray of light travelling in air is incident at grazing angle (incident angle ≅ 90°) on a slab. The point of incidence is the origin $A(0, 0)$.



Refractive index of medium = μ

$$\mu(y) = [ky^{3/2} + 1]^{1/2} \quad ...(i)$$

where $k = 1.0$ (metre)$^{-3/2}$. The refractive index of air is 1. The ray travels from air to denser medium. It continuously bends towards normal.

Slope at B is $\tan\theta$. Angle of incidence = i at B.

$\therefore \quad \frac{dy}{dx} = \tan\theta$ According to geometry, $i + \theta = 90°$

or $\theta = \frac{\pi}{2} - i \quad \therefore$ slope$\left(\frac{dy}{dx}\right) = \tan\theta$

or $\frac{dy}{dx} = \tan\left(\frac{\pi}{2} - i\right)$ or $\frac{dy}{dx} = \cot i$ ...(ii)

(b) Equation for trajectory $y(x)$ of ray in medium:

Applying Snell's law at points A and B.

$\mu_A \sin(i_A) = \mu_B \sin(i_B)$ ...(iii)

$\mu_A = 1$ from (i) because $y = 0$

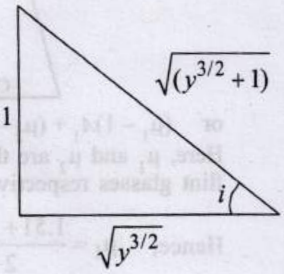
$i_A = 90°$, because of grazing incidence. $i_B = i$

$\mu_B = [ky^{3/2} + 1]$ where $k = 1.0\ m^{-3/2}$

or $\mu_B = [y^{3/2} + 1]$ ...(iv)

From (iii) and (iv), we get

$(1)(1) = [y^{3/2} + 1]\sin(i_B)$



or $\sin i = \dfrac{1}{(y^{3/2}+1)}$

or $\cot i = \sqrt{y^{3/2}}$

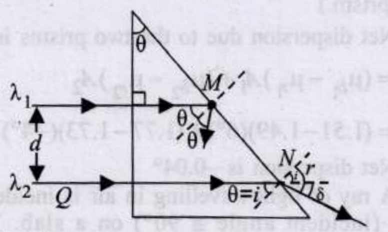
or $\cot i = y^{3/4}$

or $\dfrac{dy}{dx} = y^{3/4}$ or $y^{-3/4}\,dy = dx$

or $\int_0^y y^{3/4}\,dy = \int_0^x dx$ or $4y^{1/4} = x \Rightarrow y = (x/4)^4$

$\therefore$ Equation of trajectory of ray in medium is $y = (x/4)^4$.

26. (a) For TIR to take place, θ should be greater than C. For smaller values of C, the values of μ should be high or in other words the value of λ should be small.



From the given expression of

$$\mu(\lambda) = 1.20 + \frac{b}{\lambda^2}$$

Total internal reflection will be given by $\lambda_1 = 4000$ Å

Here, $\sin\theta = 0.8$ (given) $\Rightarrow \theta = 53.1°$

$$\therefore\quad \mu = \frac{1}{\sin\theta} = \frac{1}{0.8} = 1.25$$

$$\therefore\quad \mu = 1.2 + \frac{b}{(4000 \times 10^{-10})^2} = 1.25$$

$\therefore$ $b = 0.8 \times 10^{-14}\ m^2$

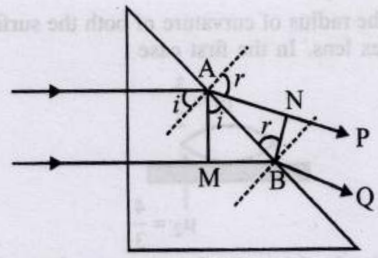
(b) Applying Snell's law at N for wavelength $\lambda_2 = 5000$Å

$$\mu = \frac{\sin r}{\sin i} \quad \text{where } \mu = 1.2 + \frac{0.8 \times 10^{-14}}{(5000 \times 10^{-10})^2} = 1.232$$

$$\Rightarrow\quad 1.232 = \frac{\sin r}{0.8} \Rightarrow r = 80.3°$$

From the figure it is clear that the deviation, $\delta = r - i = 80.3° - 53.1° = 27.2°$

(c) The intensities of transmitted beams are $4I$ and I respectively.



Path difference, $\Delta x = \mu(MB) - AN$

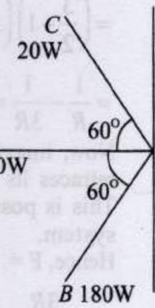
$$= \frac{\sin r}{\sin i}(AB\sin i) - AB\sin\theta = 0$$

Phase difference between rays P and Q is zero. Hence maximum intensity will be obtained from their interference.

$\therefore$ Resultant Intensity

$$I_{\max} = \left(\sqrt{I_1} + \sqrt{I_2}\right)^2 = \left(\sqrt{4I} + \sqrt{I}\right)^2 = (3\sqrt{I})^2 = 9I.$$

27. Intensity at point P, $I_P = I_A + I_B + I_C$



$$I_A = \frac{(\text{Illumination power}) \times \cos\theta}{4\pi r^2}$$

$$= \frac{90 \times \cos 0°}{4\pi \times 3^2} = \frac{10}{4\pi}\ \text{watt/m}^2$$

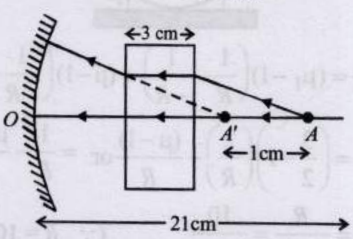
Similarly,

$$I_B = \frac{180 \times \cos 60°}{4\pi \times (1.5)^2} = \frac{10}{\pi}\ \text{watt/m}^2$$

$I_C = 20 \cos 60° = 10$

$$\therefore\quad I_P = \frac{10}{4\pi} + \frac{10}{\pi} + 10 = 13.9\ \text{W/m}^2$$

28. The rays originating from the point object at A suffers refraction due to glass slab before striking the concave mirror. For the mirror the rays are coming from A'



$$\text{Apparent shift } AA' = t\left(1 - \frac{1}{\mu}\right) = 3\left(1 - \frac{1}{1.5}\right) = 1\ \text{cm}$$

Therefore the object distance

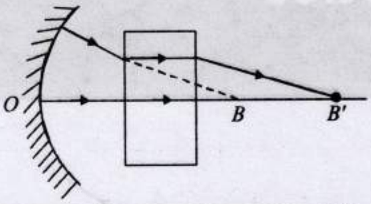
$u = OA' = OA - AA' = 21 - 1 = 20$ cm

The apparent object at A' as a virtual object for concave mirror.

Now from mirror formula, $\dfrac{1}{v} + \dfrac{1}{u} = \dfrac{1}{f}$

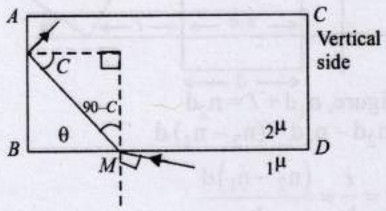
$$v = \frac{uf}{u-f} = \frac{20 \times 5}{20 - 5} = \frac{20}{3}\ \text{cm} = 6.67\text{cm}$$

The reflected rays again pass through the glass slab suffering a further shift of 1 cm. The image should have formed at B in the absence of glass slab. But due to its presence the image is formed at B'.



Therefore image distance, $OB = OB + BB'$
$= 6.67 + 1 = 7.67$ cm

29. For a grazing incident ray at BD for which $i \approx 90°$ the angle of refraction $(90° - C)$ is maximum. For this C is least. Let C is greater than the critical angle.



Applying Snell's law at M

$$_2^1\mu = \frac{\sin 90°}{\sin(90° - C)} \Rightarrow {}_2^1\mu = \frac{1}{\cos C} \quad \text{...(i)}$$

Also $${}_2^1\mu = \frac{1}{\sin C} \quad \text{...(ii)}$$

When C is the critical angle.

From (i) and (ii), $\frac{1}{\cos C} = \frac{1}{\sin C} \Rightarrow C = 45°$

$$\therefore {}_2^1\mu = \frac{1}{\sin 45°} = \sqrt{2} = 1.41$$

Therefore, the minimum value of $\mu_{glass slab} = 1.41$ for which letters on the page are not visible from any of the vertical faces of the block.